INVESTIGATIONS OF THE EFFECT OF NON - UNIFORM INSERT PITCH ON VIBRATION DURING FACE MILLING

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DEDICATED

TO MY PARENTS

Mrs & Mr K. T. MATHAI

CERTIFICATE

It is certified that the work contained in the thesis entitled INVESTIGATIONS OF THE EFFECT OF NON-UNIFORM INSERT PITCH ON VIBRATION DURING FACE MILLING, by Jose Mathew, has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

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NOMENCLATURE

M	---	diagonal mass matrix.
C	---	damping matrix.
K	---	stiffness matrix.
y	---	normal co-ordinates.
z	---	generalized co-ordinates.
ϕ	---	transformation matrix.
ξ	---	damping ratio.
ω_0	---	fundamental forcing frequency, equal to rotational speed of the cutter (0·05578)
ω_n	---	natural frequency.
$P_i(t)$	---	load vector
N_m	---	total number of nodes.
θ_0 , θ_1, θ_2, ... θ_{B-1}	---	angular positions of the teeth on the cutter blank.
B	---	total number of blades(teeth).
K	---	constant.
$f(t)$	---	tangential cutting force at the time t
N	---	cutter speed.
A_0 , B_n, C_n	---	fourier coeifficients.
A_e	---	amplitude of the spectrum.
ω	---	angular velocity.
ψ_0	---	angle at which blabe enters the work piece.
ψ_e	---	angle at which blade exits the work piece.

f_0 --- feed per tooth.

A --- reciprocal of the angular speed of the cutter.

ABSTRACT

In metal cutting process, vibration is one of the major phenomenon which affects the surface finish, tool life, and quietness of the working environment. Therefore, vibration control has always been played an important role in obtaining good machining performance.

In face milling, optimal tooth spacing via spectral redistribution criteria has been used as one of the method for vibration control. The effect of such a type of cutter with non-uniform pitch on machine tool vibration is studied in the present work.

When the dynamic frequency response of a machine tool work -piece system is known, a special purpose cutter can be designed to minimize the vibration in the cutting zone for a particular range of cutting speed.

The frequency response spectrum of the machine tool is found out analytically using finite element program. The structure of the machine is idealized with beam elements and the mass of the machine is lumped appropriately at different nodes. The spacing between the teeth is found out by minimizing the total power of the relative cutter work-piece vibration.

The newly designed cutter is fabricated and tested experimentally to find out its effectiveness when compared with standard cutter with evenly spaced inserts.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Face milling, being a process with a relatively high rate of metal removal, is one of the most widely used machining processes. In machining processes, vibration is a major phenomenon which affects the surface finish and the tool life. Therefore vibration control has played a major role in obtaining good machining performance.

In face milling operations, two factors, namely cutting force and chatter, causes the cutting system to vibrate. Cutting forces always result in forced vibrations and chatter, a kind of self excited vibration, occurs under certain conditions, usually at heavy cuts .

Vibration problems in face milling are usually dealt with three general approaches.

1. Designing a more rigid Machine-tool-Fixture-workpiece system (MTFW).
2. Selection of proper cutting conditions.
3. Designing cutters for optimal tooth spacing.

The effectiveness of these approaches depend on a host of factors. In many cases machine tool and fixture already exist. Also, there may be a difficulty in setting the cutting conditions

at the optimum level. Hence, demands for high productivity suggest that the manipulation of tooth spacing may be a useful approach to the suppression of vibrations.

Basically, two criteria have been used in finding out the optimal tooth spacing.

1. stability criterion, and
2. force spectral redistribution criterion.

The stability criterion uses uneven spacing to disturb the regeneration effect and obtain the maximum system stability. The variable spacing employed is generally some fixed pattern, for example , linear variation of the pitch or a repeating pattern of two different pitches. The criterion of spectral distribution uses uneven spacing to redistribute the frequency components in the cutting force signals to avoid the system from exciting at the natural frequencies. The aim of the present work is to design a cutter of odd number of teeth with optimum blade spacing so that the forced vibration in a face milling operation is minimized.

1.2 REVIEW OF LITERATURE

Jan Slavicek [1] has reported some favourable effects of the irregular pitch of milling cutter teeth on the stability of the cutting process. This effect is caused by the influence of speed, chatter-frequency and tooth pitch on the regeneration of chatter. He has based his study on the theory of chatter developed by Tlusty and Polacek [2]. However, the study was limited to 'pitch periods' of the two cutting edges, thereby necessitating a pitch ratio for each combination of cutting speed and natural frequency.

Vanherck [3] has demonstrated that by increasing the number of cutting edges in one 'pitch period', a stability-gain can be achieved over a large cutting speed range. Through simulation of the actual conditions on a computer, he has established that by a suitable selection of 'pitch ratio' and 'pitch period' , an increased stability can be maintained over a wide range of cutting speeds and natural frequencies.

Sudheendra and Shanbouge [4] have reported a comparative study of the effectiveness of unequally spaced cutters with linear-pitch-evolution to that of cutters with equal pitch, over different cutting speeds, width of cut and various natural frequencies. The experimental results indicate that the productivity of the milling cutters could be considerably improved by incorporating an optimum unequal spacing of teeth. It has also been reported that an odd numbered teeth cutter gives a slight improvement in marginal stability gain over the even numbered teeth cutters at certain frequencies.

Doolan et.al [5], have developed a method to choose the blade spacing for a face milling cutter that will minimize vibration. They have designed a special purpose cutter which minimizes the relative vibration in the MTFW system for a particular cutting speed. Here the dynamic frequency response of the MTFW system was known. It was assumed that the tangential cutting force of a blade can be approximated by an impulse.A systematic trail and error method using geometric modulation was proposed to evolve a set of blade spacing that would reduce the vibration.

An improved model for the cutting process has been described by Doolan, Wu [6]. The nature of the tangential cutting force of a blade was approximated by a rectangular pulse instead of an impulse, and it was experimentally verified. The height of the impluse is related to the blade spacing.For an unevenly spaced cutter, the feed per teeth, and thereby the tangential cutting force is different on different blades. Therefore a model was built to describe the variation of the cutting force with feed. The developed model for cutting force is a function of feed, depth of cut and speed. But here, it has been stated that the cutting speed has not much influence on the cutting force. Minimization of vibration as a function of blade spacing has been obtained by means of a non linear regression routine and a random search procedure.

In practice, there may be situations where all the operating conditions are not known and the MTUF frequency response is unknown. This was taken care of by Wu et. al. [7] while designing a multipurpose cutter. It has been reported that this new method effectively controls the vibration levels under all operating conditions.

Fu et. al. [8], has developed a 'mechanistic' model for the prediction of the forces in an MTFU system. This model predicts the force system in face milling over a range of cutting conditions, different cutter geometries, work pieces and processes, including different positions of the cutter with respect to the work piece.

A new approach, based on minimum vibration that considers

both stability and forced vibration simultaneously, was developed by Kapoor et.al [9]. They developed the new criteria through a non-linear optimization method in the dynamic force model. The model was of a 'closed loop' type which consisted of a cutting process, strutural dynamics and a feed back mechanism. Chatter and forced vibrations were analyzed from the vibration contour maps. It has been reported that optimal spacing as determined by the vibration criteria not only increases the system robustness against chatter, but also minimizes the forced vibration at a prespecified set of cutting conditions.

Various finite element techniques are used to get the frequency response spectrum, since the experimental techniques available are time consuming and expensive. By using the 'classical beam theory ', Maltback [10] calculated the natural frequencies and mode shapes of a radial drilling machine structure. Taylor and Tobias [11] studied the application of the finite element technique to represent the structural parts of a radial drilling machine arm and a lathe. Static and dynamic analysis of a planing machine was done by Taylor [12]. The lumped parametric method was used in the above two works to generate the mass matrix. By idealizing the structure with frame elements, Cowley and Fawcett [13] analyzed a plano-milling machine to determine the static deflections, natural frequencies and mode shapes. Kainth and Moorthy [14] also used the lumped parametric method to predict the static deflections, natural frequencies and the mode shapes of the arm of a radial drilling machine.

In order to predict the static and dynamic characteristics of

a machine tool structure accurately and consistently, the structure has to be represented by a suitable model. The model must also be simple enough from a computational point of view. Rao, S.S and Reddy, C.P. [15] idealized the milling machine model with triangular plate elements and frame elements. The model developed has been found to be an efficient one. The frame elements were used to model the ribs of the overarm, the overarm joint with the column, the arbor and the arbor support. The main members of the column, the overarm and table were modelled with triangular plate elements.

Rao, S.S. and Ramana, G.V. [16] used the finite element displacement method to model the machine tool structures. They have also compared their model with the other available models. Space frame elements were used here to idealize the column and arm of a radial drilling machine. The triangular plate elements were used to idealize the column and overarm of the plano-milling machine, since these members have sufficient thickness and width while the space frame elements were used to represent the cross-slide. In dynamic analysis, the eigen value problem was solved by using the Rayleigh-Ritz sub space iteration algorithm, which is considered to be one of the most efficient solution techniques.

Lee et.al. [17] used the standard SAP-IV [structural analysis programme] finite element program to perform the structural analysis. It has an element library of struss, beam, plate, shell, brick, boundary and pipe elements which is adequate for almost all structural modellings. After the calculations of stiffness, mass

and load matrices, the SAP-IV program performs static analysis, dynamic response analysis and computes the eigen values.

1.3 OBJECTIVE OF THE PRESENT WORK

The literature review reveals that many researchers have studied the problem of minimizing the vibration during milling by either using an odd number of cutters or unevenly spacing the cutting teeth. The objective of the present work is to investigate the effect of non-uniform insert pitch cutters on vibration during face milling.A milling cutter is planned to design and which minimizes the relative vibration between the cutter and the work piece.The cutter uses odd number of teeth which are unevenly spaced along the cutter periphery. The optimum spacing between the cutters is planned to determine with the help of a standard optimization program, NAG. (Numerical Algorithm Group). As mentioned eariler, to avoid a time consuming experimental procedure, the present work utilizes the finite element techniques using the standard structural analysis programme, SAP-IV, to determine the natural frequencies of the Machine Tool Sturcture. Furthermore, the cutter which is designed and fabricated is planned to be tested experimentally to show its effectiveness in reduction of vibration when compared with the evenly spaced standard cutters.

1.4 ORGANIZATION OF THE THESIS

Chapter 2 deals with the finite element method used to determine the frequency response spectrum of the milling machine.

Chapter 3 describes the theory used to develop the cutting force models. The theoritically obtained vibration spectra of the optimum spacing cutter are analysed and compared with the standard

available evenly spaced teeth cutter.

Chapter 4 describes the experimental setup used. The design details of the newly developed cutter are also explained here.

In Chapter 5, the experimental results obtained with the newly designed cutter and that of the standard cutter are compared. The conclusions and suggestions for future work are also explained here.

CHAPTER 2

RESPONSE SPECTRUM OF THE MILLING MACHINE

2.1 NECESSITY FOR ANALYTICAL METHODS

Experimental techniques to find out the frequency response of a machine require expensive and complicated setups, and therefore analytical methods are often resorted to in practice. In the present work, a finite element analysis using beam elements has been carried out to determine the natural frequencies of the machine. The mass of the machine including those of the inner components have been lumped at appropriate nodes. Natural frequencies and mode shapes so obtained are used to compute the frequency response of the machine. A computer programme was developed for calculating the response of the machine at the cutterpoint for various forcing frequencies.

2.2 DETERMINATION OF NATURAL FREQUENCIES OF THE STRUCTURE BY FINITE ELEMENT METHOD.

The details of the milling machine on which the study was conducted as shown in Figure 2.1. Initially the machine was idealised for finite element studies using 13 beam elements as shown in fig.2.2 .The cross section of the different parts of the machine is shown in fig.2.5. The Structural Analysis Programme (SAP IV) was used to analyse the structure. The mass of the structure and those of inner components were appropriately lumped at various nodes. The natural frequencies corresponding to various modes are given in Table 2.1. The analysis was performed by increasing the number of elements to 26 (fig 2.2) and then to

50 (fig 2.3) and the natural frequencies obtained corresponding to each mode are given in Table 2.2 and 2.3. It was found that with 50 elements the eigen values converged with reasonable accuracy and hence the natural frequencies obtained with 50 elements have been taken as the natural frequencies of the machines. The elements of eigen vector (ϕ_{ij}) for various natural frequencies of the machine corresponding to the cutter point is given in Table 2.4.

2.3 DETERMINATION OF FREQUENCY RESPONSE SPECTRUM OF THE MILLING MACHINE.

The equation of motion of the damped system is given as

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{F(t)\} \qquad \ldots(2.1)$$

where,

[M] : diagonal mass matrix

[C] : symmetric damping matrix

[K] : symmetric stiffness matrix

y : normal coordinates

It is known that undamped linear systems possess normal modes. However, it is not necessarily so in the case of damped systems. The damped systems will have normal modes, when the damping matrix is a linear combination of the stiffness and intertia matrices. A necessary and sufficient condition for a damped system to possess normal modes is that the damping matrix be diagonalized by the same transformation which uncouples the undamped system.

Introducing the transformation matrix

$$\{y\} = [\phi]\{z\} \qquad \ldots(2.2)$$

where $[\phi]$ is the modal matrix obtained in the solution of undamped vibrating systems, and z is the generalised co- ordinates.

Substituting the eqn. 2.2 and its derivatives in to eqn. 2.1 leads to

$$[M]\ [\phi]\ \{\ddot{z}\} + [C]\ [\phi]\ \{\dot{z}\} + [K]\ [\phi]\ \{z\} = \{F(t)\}. \qquad ...(2.3)$$

Premultipling the eqn.2.3 by the transponse of the modal vector yields,

$$\{\phi\}_i^T\ [M]\ [\phi]\ \{\ddot{z}\} + \{\phi\}_i^T\ [C]\ [\phi]\{\dot{z}\} + \{\phi\}_i^T\ [K]\ [\phi]\ \{z\} = \{\phi\}_i^T\ \{F(t)\} \qquad ...(2.4)$$

It is noted that the orthogonality property of the modal shapes,

$$\{\phi\}_i^T\ [M]\ [\phi]_m = 0, \qquad ...(2.5)$$

$$\{\phi\}_i^T [K]\ [\phi]_m = 0, \quad m \neq i \qquad ...(2.6)$$

causes all components except the i^{th} mode in the first and third terms of eqn. 2.4 to vanish. A similar reduction is assumed to apply to the damping term in eqn.2.4,i.e.,it is assumed that,

$$\{\phi\}_i^T\ [C]\ \{\phi\}_m = 0,\ i \neq m. \qquad ...(2.7)$$

then the coefficient of the damping term in eqn 2.4 will reduce to $\{\phi\}_i^T\ [C]\ \{\phi\}_i$

In this case eqn. 2.4 can be written as

$$M_i\ddot{z}_i + C_i\dot{z}_i + K_i z_i = F_i(t) \qquad ...(2.8)$$

or, alternatively as

$$\ddot{z}_i + 2\xi_i\omega_{ni}\dot{z}_i + \omega_{ni}^2\ z_i = F_i(t)/\ M_i \qquad ...(2.9)$$

in which case

$$M_i = \{\phi\}_i^T\ [M]\ \{\phi\}_i, \qquad ...(2.10)$$

$$K_i = \{\phi\}_i^T\ [K]\ \{\phi\}_i = \omega_{ni}^2\ M_i\ , \qquad ...(2.11)$$

$$C_i = \{\phi\}_i^T\ [C]\ \{\phi\}_i = 2\xi_i\omega_{ni}M_i, \qquad ...(2.12)$$

wher, ω_{ni} is the natural frequency at the i^{th} mode.

and ξ_i is the damping ratio at the i^{th} mode.

$$F_i(t) = \{\phi\}_i^T\ \{F(t)\} \qquad ...(2.13)$$

$$\{\phi\}_i^{\ T}\ [M]\ \{\phi\}_{\ i} = 1 \qquad ...(2.14)$$

If we use normalised model vector for trasformation, M_i will be a unity i.e.,

$$M_i = 1 \qquad ...(2.15)$$

so that eqn.2.9 reduces to

$$\ddot{z}_i + 2\xi_i\omega_{ni}\dot{z}_i + \omega_{ni}^2\ z_i = F_i(t) \qquad ...(2.16)$$

which is a set of N uncoupled equations ($i = 1,2,...,N$)

The amplitude of steady state response of the system in the

generalised co-ordinates in the i^{th} mode is given by

$$Z_i(t) = \frac{F_i(t)}{\left[\left(\omega_{ni}^2 - \omega^2 \right)^2 + \left(2 \xi_i \omega \omega_{ni} \right)^2 \right]^{1/2}} \qquad \ldots(2.17)$$

Since the dynamic force acts only at the node(j^{th} node) corresponding to the cutter point, the forcer vector $F_i(t)$ contains only one term and hence F_i in the equation 2.17 is given by,

$$F_i = \phi_{ij} P_j \qquad \ldots(2.18)$$

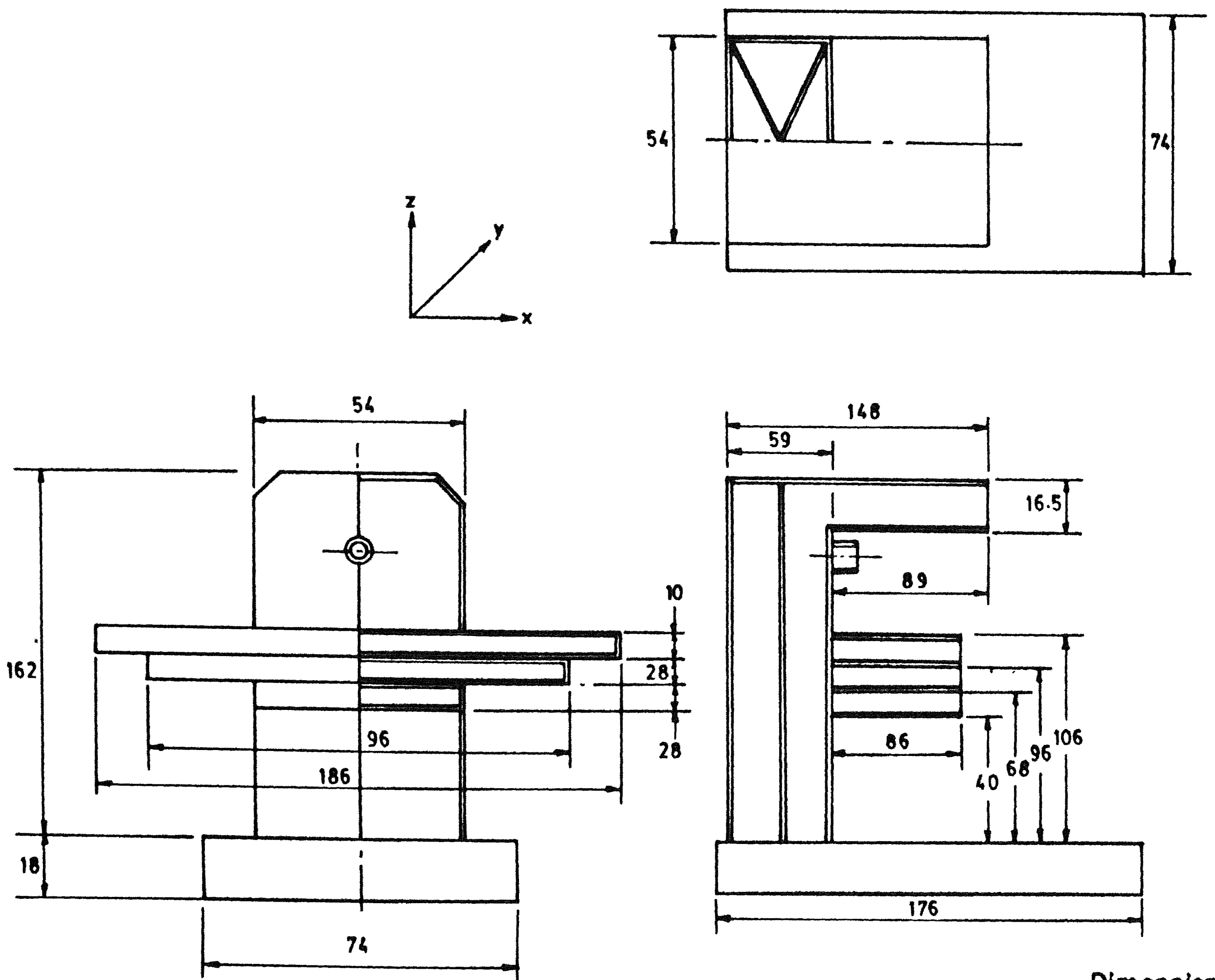
where P_j is the amplitude of the dynamic force acting on the cutter. Hence the displacement of the mode at the cutter-point can be obtained as

$$y_j = \sum_{i=1}^{N_m} \frac{\phi_{ij}^2 P_j}{\left[\left(\omega_{ni}^2 - \omega^2 \right)^2 + \left(2 \xi_i \omega \omega_{ni} \right)^2 \right]^{1/2}}$$

where, N_m is the total number of modes

$$\ldots(2.19)$$

The response of the machine at the cutter point can be obtained from the eqn.2.19. The different values corresponding to P_j equals to unity is calculated with the help of the computer for a range of forcing frequency and the results are plotted in fig.2.6.



Dimensions in cm
Plate thickness 1.7cm

FIG. 2.1 MILLING MACHINE STRUCTURE

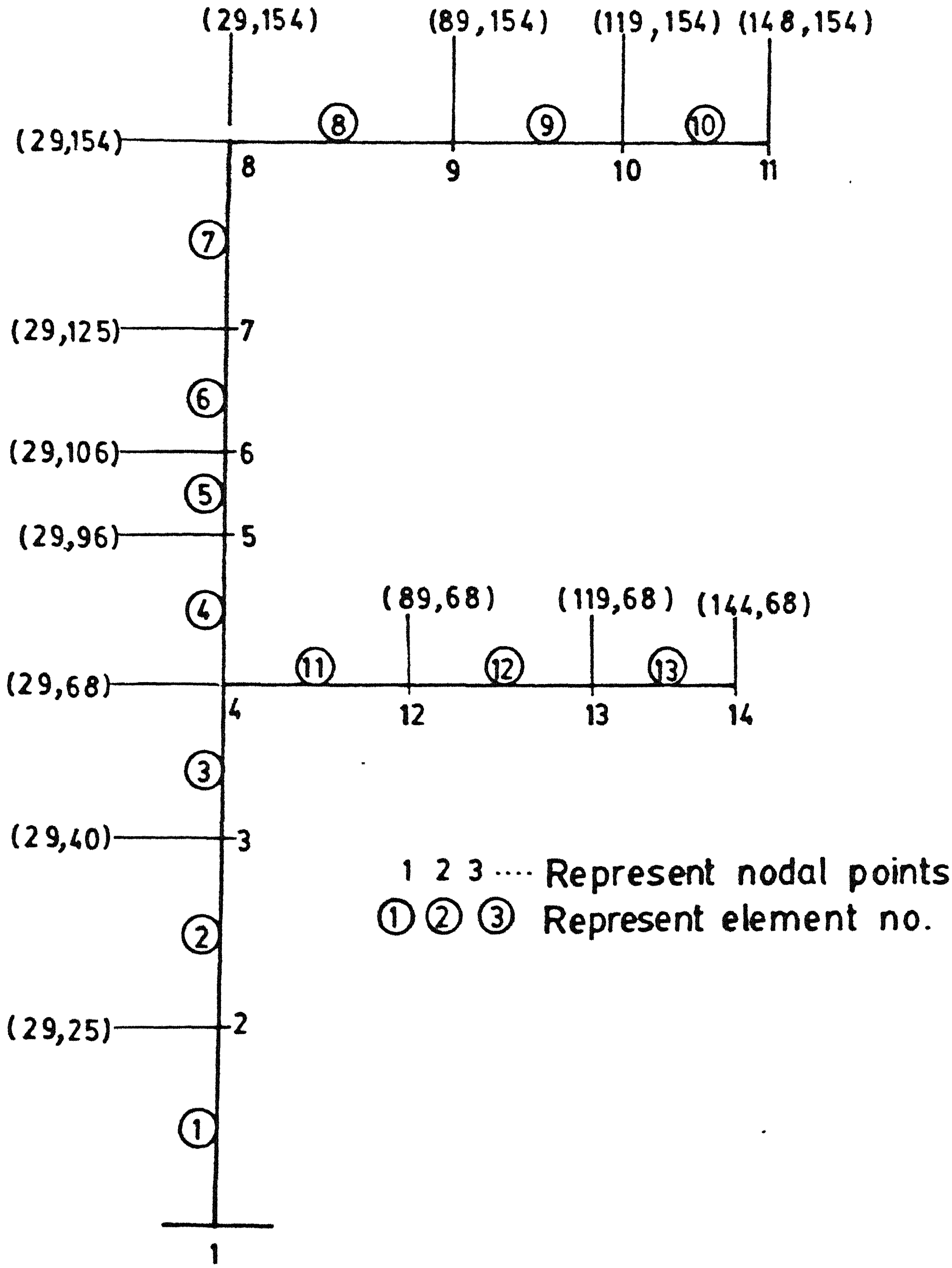


FIG. 2·2 MILLING MACHINE FINITE ELEMENT IDEALIZATION - 1 (13 Element)

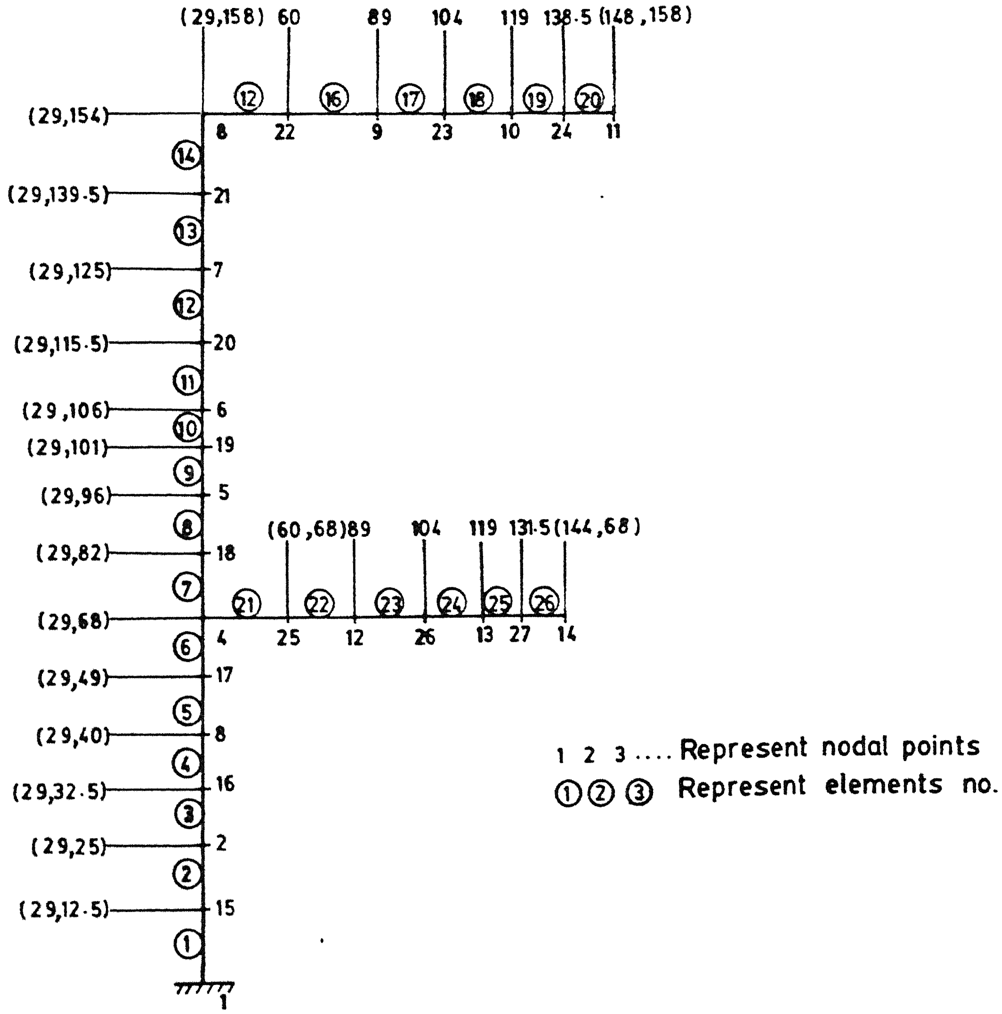


FIG. 2.3 MILLING MACHINE FINITE ELEMENT IDEALIZATION 2
(26 Elements)

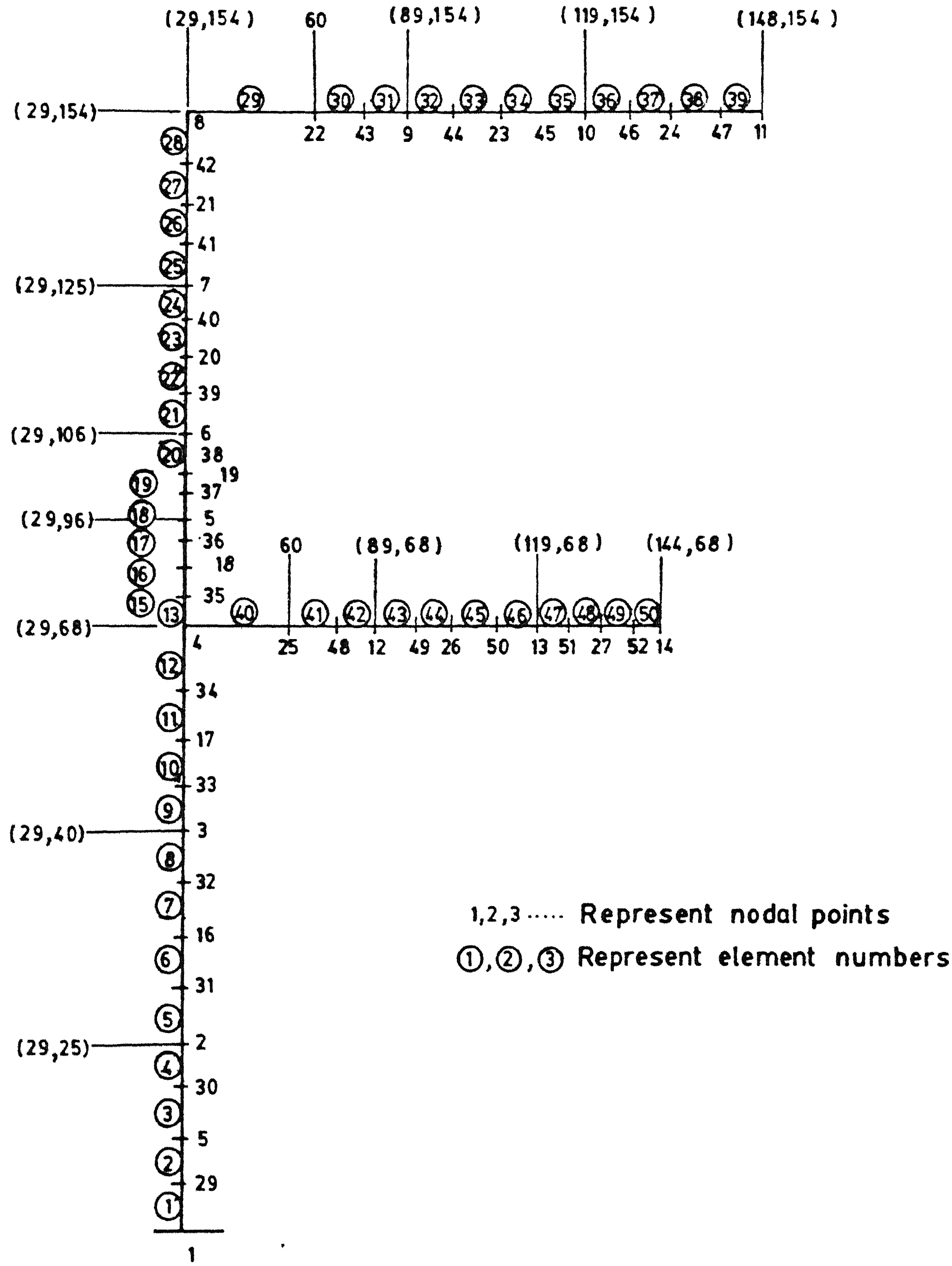


FIG. 2·4 MILLING MACHINE FINITE ELEMENT IDEALIZATION 3 (50 Elements)

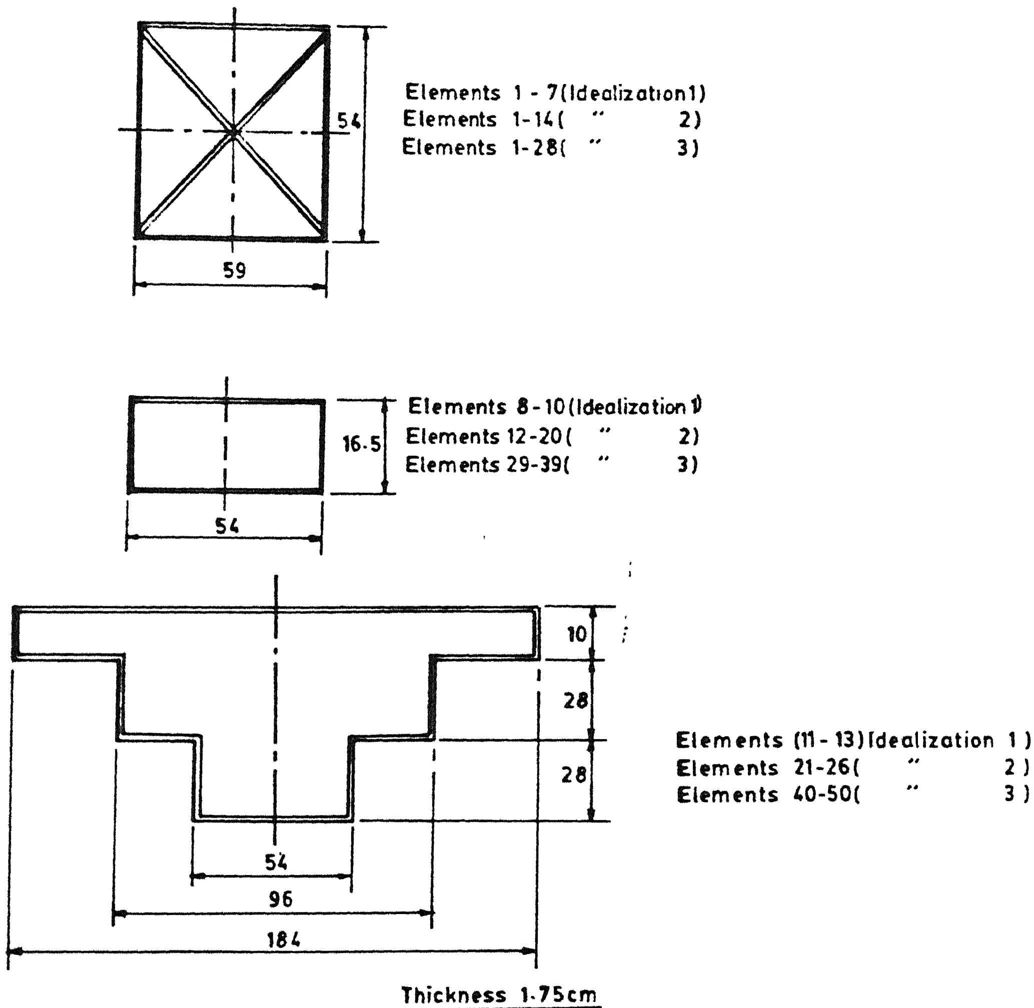


FIG. 2.5 CROSS SECTION OF MILLING MACHINE STRUCTURE

Mode mumber	Natural frequencies (rad/sec)
1	268
2	493
3	669
4	1756

Table 2.1 Natural frequencies of the milling machine structrue with 13 elements

Mode mumber	Natural frequencies (rad/sec)
1	264
2	486
3	656
4	1746

Table 2.2 Natural frequencies of the milling machine structure with 26 beam elements

Mode mumber	Natural frequencies (rad/sec)
1	251
2	463
3	642
4	1687

Table 2.3 Natural frequencies of the milling machine structure with 50 beam elements.

Mode number	Element of eigen vector corresponding to cutter point (Y-direction)
1	0.09386
2	0.25770
3	0.06149
4	-0.3757E-17

Table 2.4 Elements of eigenvector corresponding to cutter point for various modes (with 50 elements)

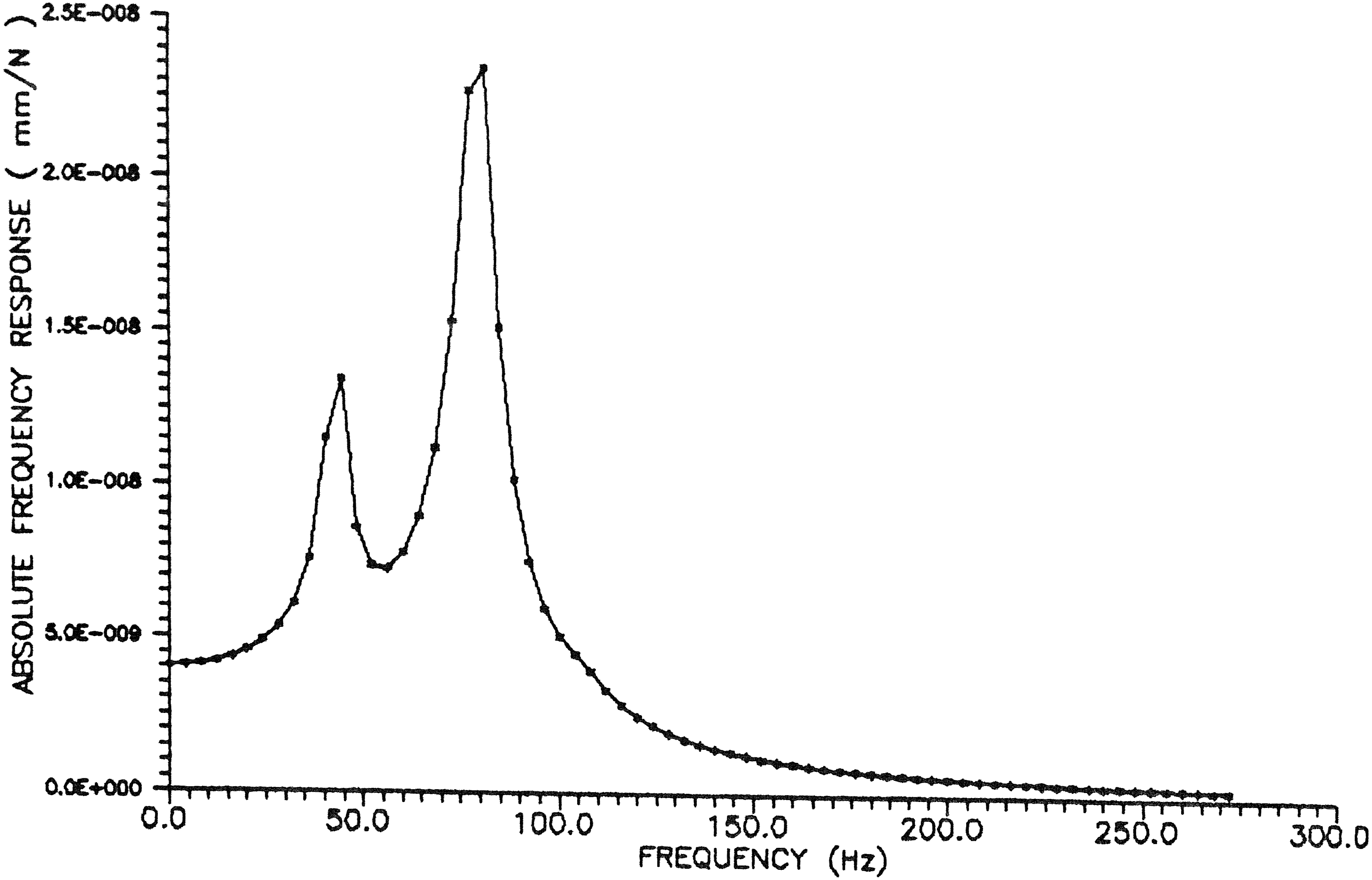


Fig. 2.6 Response Spectrum of the Milling Machine.

CHAPTER 3

DETERMINATION OF OPTIMUM BLADE SPACING

3.1 INTRODUCTION

To study the effect of non uniform spacing of teeth on vibration during face milling, two models have been analysed.In the first model, the tangential cutting force is approximated by as impulse force where as in the second model, it is approximated by a rectangular pulse.

3.2 MODEL 1

A special purpose cutter is designed to minimize the relative vibration between the cutter and work piece for a particular cutting speed, when the dynamic frequency response of the machine-tool-fixure-work piece (MTFW) system is known. The model approximate the excitation force due to impacting blades to an impluse force. In the present model the tangential cutting force of the blade is approximated by an impulse force so that the amplitude of the forcing function can be computed using fourier series analysis. The theory can be summarised as follows.

3.2.1 ASSUMPTIONS:

The following assumptions are made in developing the theory.

1. The impact of each blade on the work piece is instantaneous.
2. The impact amplitude is the same for each blade and

is uneffected by changes in the blade spacing.

Let θ_0, θ_1, $\theta_2,\ldots,\theta_{B-1}$ be the anglar positions of the blades in a cutter having B number of blades, as shown in fig. 3.1. θ_0 may arbitrarily be taken to be zero. The impact force of the blades on the work piece may be characterized by [ref.5]

$$f(t)=\begin{cases} K & t = (2\pi m + \theta_i)A \quad m = 0,\pm1,\pm2,\ldots \\ 0 & \text{for all other } t \quad i = 0,1,2,\ldots\ B-1 \end{cases} \qquad \ldots(3.1)$$

where,

K is a constant, and

A is the reciprocal of the angular speed of the rotation of the cutter; $A = 60/2\pi N$, N being the speed of the cutter in rpm.

The functin f(t) is seen to be periodic with period $2\pi A$. The fourier series expansion of f(t) is given by

$$f(t)= A_0 + \sum_{n=1}^{\infty} B_n \operatorname{Sin}\left[\frac{nt}{A}\right] + \sum_{n=1}^{\infty} C_n \operatorname{Cos}\left[\frac{nt}{A}\right] \qquad \ldots(3.2)$$

where,

$$A_0 = \frac{1}{2\pi A}\int_0^{2\pi A} f(t)dt = \frac{KB}{2\pi A} \qquad \ldots(3.3)$$

$$B_n=\frac{1}{\pi A}\int_0^{2\pi A} f(t) \operatorname{Sin}\left[\frac{nt}{A}\right] dt = \frac{K}{\pi A}\sum_{i=0}^{B-1} \operatorname{Sin}(n\theta_i) \qquad \ldots(3.4)$$

$$C_n=\frac{1}{\pi A}\int_0^{2\pi A} f(t) \operatorname{Cos}\left[\frac{nt}{A}\right] dt = \frac{K}{\pi A}\sum_{i=0}^{B-1} \operatorname{Cos}(n\theta_i) \qquad \ldots(3.5)$$

The power of the forcing function, $f(t)$ at the n^{th} harmonic of the cutter speed is given by

$$S_n = \left[B_n^2 + C_n^2 \right] \qquad \text{...(3.6)}$$

and the amplitude spectrum is given by

$$A_e(n\omega_0) = A_n = (S_n)^{1/2} \qquad \text{...(3.7)}$$

Knowing the dynamic frequency response function of a particular MTFW system ,the relative vibration between the cutter and the work piece can be written as

$$S_R (\omega) = A_e(\omega) \times S_s(\omega) \qquad \text{...(3.8)}$$

where,

$S_R(\omega)$ is the amplitude of the realative vibration,

$A_e(\omega)$ is the amplitude spectrum of the exitation force,

$S_s(\omega)$ is the amplitude spectrum of the MTFW system.

The total power corresponding to the resulting relative vibration between the cutter and work piece is given by

$$R(\theta) = \sum_{k=1}^{H} S_R^2 (\omega_0 k) \qquad \text{...(3.9)}$$

where,

ω_0 is the fundamental frequency of the forcing function ,equal to the rotational speedof the cutter.

H is the frequency range beyond which $S_s(\omega)$ is small.

The objective is to minimize the total power of the relative vibration R(θ), (Eqn.3.9). More specifically one can minimize vibration in either X, Y, Z directions,[ref.fig.2.1], or the resulting vibration, or some function of all the three directional vibrations. Considerations in choosing among these alternatives could include the relative cutting force magnitudes,the relative system stiffness and the natural frequencies of the machine in the three directions.

As the tangential dynamic cutting force in the MTFW system for study is in the Y-Z plane, the dynamic response in the X direction is ignored. Due to the large stiffness in the Z direction the frequency response in the Y direction is more prominent than that of the Z direction. Hence, in this work, the the objective is to minimize the sum of the vibrations in the Y direction.

Minimization of the vibration power [R(θ)] as function of the blade spacing gives the optimum cutter design.The non- linear objective function [R(θ)] is minimized by utilising the standard subroutine for optimization from the library of NAG(Numerical Algoritm Group), subject to the following constraints on the design variables which arise out of practical and geometrical considerations.

1. Geometrical constraints

$$0 \leq \theta \leq 360 \text{ degrees.}$$

2. $\theta_{i+1} - \theta_i \geq 45$ degrees for the first six blades and $360 + (\theta_1 - \theta_7)$ for the last blade.

3.2.2 DESIGN OF THE CUTTER

A new cutter is designed according to the above theory. The cutting parameters are taken in such a way that it will satisfy the prescribed conditions for machining the chosen work-piece with the standard available cutter . Cutting speeds of 200,240, 315 and 400 rpm are selected for investigation.The optimization programme is run at different initial values of the variable(blade angles).The number of the inserts are chosen as seven as per the design constraints.

In the analysis,it has been seen that objective function is the lowest at the cutting speed of 315 rpm. Some typical results ,where the values of the objective function is the minimum is given in Table 3.1.The corresponding values of the blade spacings are also given.It has been observed that here, the final values of the objective functions are more or less remains the same.Even though the angular positions are different,the angular spacings are remains the same.A representative figure which shows the the different angular positions and the angular spacings are given in fig 3.3.

3.3 MODEL 2

In this model the cutting force of a single blade is approximated by a pulse of duration equal to the duration of cut and height of the pulse is determined by the cutting conditions(speed,feed,and depth of cut).

3.3.1 ASSUMPTIONS:

The following assumptions are made in this theory.

1) The cutting force is approximated by a rectangular pulse whose height is governed by the blade spacing.

2) For an unevenly spaced cutter, the feed per tooth and hence the cutting force is different on different blades.

The accepted model of the cutting force f (from experimentation) is as follows-[ref.6]

$$f = \alpha_0 \; f_o^{\alpha_1} \; d^{\alpha_2} \; N^{\alpha_3} \qquad \ldots(3.10)$$

here α_0, α_1, α_2, α_3 are constats.

The experimentally determined confidence limits of the different parameters are as follows.

$$0.673 \le \alpha_1 \le 0.817$$

$$0.611 \le \alpha_2 \le 0.717$$

$$-0.143 \le \alpha_3 \le 0.045$$

It can be observed that α_3 is very close to zero, which indicates that the cutting speed has less influence on the cutting force compared to other parameters. The total force due to the cutting force on the cutter with 'B' number of teeth is given by,

$$f(t) = \sum_{i=0}^{B-1} f_i(t) \qquad \ldots(3.11)$$

where

$f_i(t)$ is the force due to the i^{th} blade.

$$f(t) = \begin{cases} K\left[\left(\dfrac{\theta_i - \theta_{i-1}}{2\pi/B}\right)f_0\right]^{\alpha_1}, & (2\pi m + \psi_0 + \theta_i)A \le t \le (2\pi m + \psi_e + \theta_i)A \\ 0, \text{ otherwise.} & \end{cases}$$

$$m = 0, \pm 1, \pm 2, \ldots$$

...(3.12)

where ψ_0 - angle at which blade enters the work piece

ψ_e - angle at which blade exits the work piece

f_0 - feed per tooth

A - reciprocal of the angular speed of the rotation of the cutter

K - is a constant

The fourier series representation of f(t) is given by

$$f(t)=f_0+\sum_{n=1}^{\infty} B_n \sin(\frac{nt}{A})+\sum_{n=1}^{\infty} C_n \cos(\frac{nt}{A})$$

...(3.13)

$$f_0 = \frac{1}{2\pi A} \int_0^{2\pi A} f(t)dt = \frac{K'\psi}{2\pi A} \sum_{i=0}^{B-1} (\theta_i - \theta_{i-1})^{\alpha_1}$$

...(3.14)

$$B_n = \frac{1}{\pi A} \int_0^{2\pi A} f(t) \sin (\frac{nt}{A})dt$$

$$= \frac{K'}{\pi n} \sum_{i=0}^{B-1} (\theta_i - \theta_{i-1})^{\alpha_1} \left[\mathrm{Cos}(n [\psi_e + \theta_i]) - \cos(n [\psi_0 + \theta_i])\right]$$

...(3.15)

and,

$$C_n = \frac{1}{\pi A} \int_0^{2\pi A} f(t) \cos (\frac{nt}{A})dt$$

$$= \frac{K'}{\pi n} \sum_{i=0}^{B-1} (\theta_i - \theta_{i-1})^{\alpha_1} \left[\sin(n [\psi_e + \theta_i]) - \sin(n [\psi_0 + \theta_i])\right]$$

...(3.16)

where $K' = K(f_0 B/2\pi)^{\alpha 1}$

The amplitude of the n^{th} harmonic of the spectrum of the exitation force is given by

$$A_e(n\omega_0) = A_n = (B_n^2 + C_n^2)^{1/2} \qquad ...(3.17)$$

If $S_s(\omega)$ denote the MTFW frequency response function,Then the amplitude of the relative vibration between the cutter and the work piece is given by

$$S_R(\omega) = A_e(\omega) \times S_s(\omega) \qquad ...(3.18)$$

Then total power of the relative vibration , $R(\theta)=\sum S_R^2(n\omega_0)$ ...(3.19)

Minimization of the vibration power $[R(\theta)]$, which is a function the blade spacing gives the optimum cutter design. As described in Model 1, the minimization is obtained by non-linear minimization procedure with the help of subroutine in NAG library.

3.3.2 DESIGN OF THE CUTTER

As per the second model the tangential cutting force of a blade is also depended on the entrance angle and the exit angle. A diagram showing the blade entrance and blade exit angles is given in fig.3.2. The theory is developed by using an entrance angle of 50 degrees and exit angle of 130 degrees,which is close to values used in experiments.The minimization programme is run at the various cutting speeds ,namely 200,240,315,400 rpm and at various feeds ranging from 0.010 mm/tooth to 0.030mm/tooth. The details of the speed and feed ranges chosen are given below.

Speed 200 rpm	feed 0.010 mm/tooth
	feed 0.015 mm/tooth

	feed 0.020 mm/tooth
	feed 0.030 mm/tooth
Speed 240 rpm	feed 0.010 mm/tooth
	feed 0.015 mm/tooth
	feed 0.020 mm/tooth
	feed 0.030 mm/tooth
Speed 315 rpm	feed 0.010 mm/tooth
	feed 0.015 mm/tooth
	feed 0.020 mm/tooth
	feed 0.030 mm/tooth
Speed 400 rpm	feed 0.010 mm/tooth
	feed 0.015 mm/tooth
	feed 0.020 mm/tooth
	feed 0.030 mm/tooth

The analysis shows that the objective function is the minimum corresponding to the cutting conditions of speed 315 rpm and feed 0.010 mm/tooth. Table 3.2 shows some of the typical values of blade spacings obtained corresponding to the minimum value of the objective function.The results obtained reveals that the angular spacings for the optimum cutter remains the same as that of the model 1. The figure which represents the different angular positions are shown in fig.3.4.

3.4 ANALYSIS OF THE THEORITICAL RESULTS

Two cutter spacings were selected from the above two model results corresponding to the lowest values of $R(\theta)$.The cutter spacings selected for experimental verifications are,

1. angular positions(0 - 45 - 112 - 158 - 225 - 270 - 315 degrees) ----design 1 -(fig.3.5.1)
2. angular positions (0 - 45 - 90 - 135 - 202 - 248 - 293 - degrees) ----design 2 - (fig.3.5.2)

The vibration spectra obtained with the theoritcal analysis for the standard cutter are given in fig 3.6 for model 1, and fig.3.10 for model 2. Fig 3.7 and fig.3.11 give the theoritcal vibration spectra of the evenly spaced cutter with odd number of inserts(7 inserts) corresponding to model 1 and model 2. The vibration spectra for the newly designed cutters can be obtained from fig 3.8 and fig.3.9 for the model 1 .Fig 3.12 and fig 3.13 give The vibration spectrum for the newly developed cutters for the model 2.

For comparision purpose, a table has been prepared (Table 3.3).It will give the percentage reduction of the vibration power with respect to that of the evenly spaced eight number of inserts cutter in different cases. It has been seen that a reasonable percentage of reduction in vibration is obtained in the two selected cutters.Both the designs give more than 50 percentage reduction in the two models.

A comparitive table for analysing the percentage reduction of the maximum amplitude of the relatve vibration is shown in table 3.4. It can been seen that the maximum amplitude of the vibration is decreased considerably in the two designed models.

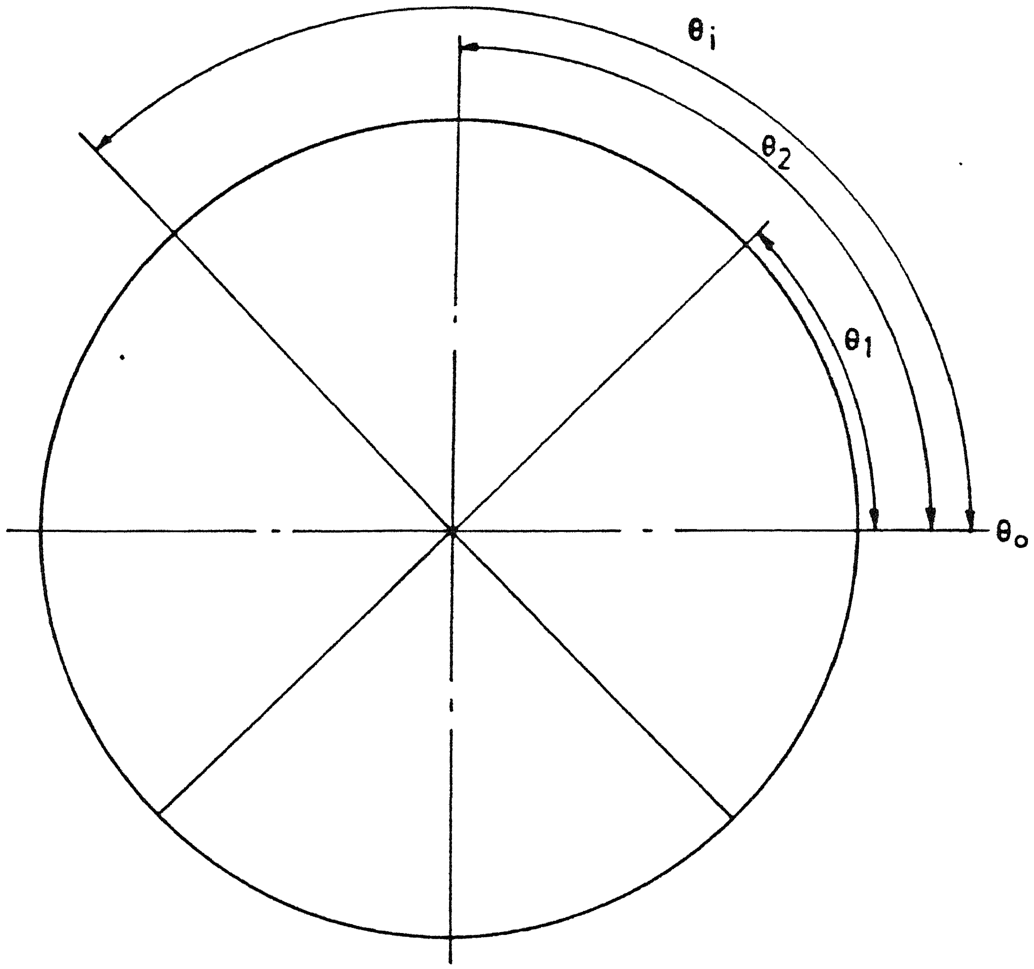


FIG. 3.1 LOCATIONS OF CUTTER BLADES

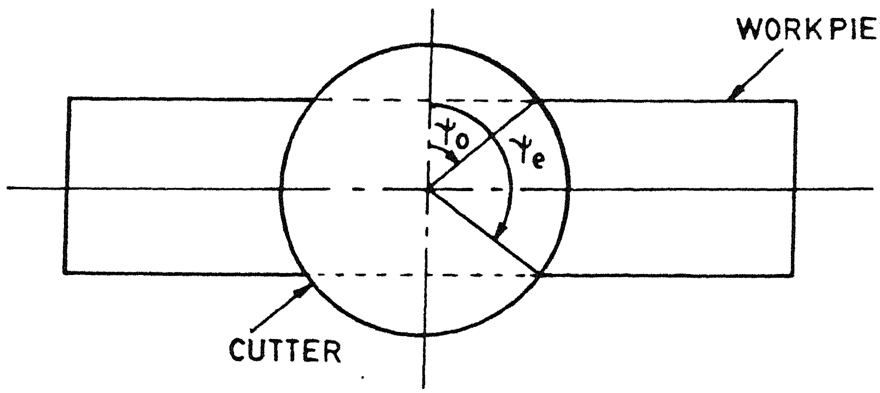


FIG. 3.2 A DIAGRAM SHOWING THE BLADE ENTRANCE AND BLADE EXIT ANGLE

	Blade angles							R(θ)
starting value	0.0	51.5	103.0	154.5	206.0	257.5	309.0	600.0
converging value	0.0	45.0	90.0	157.0	203.0	270.0	315.0	143.0
starting value	0.0	50.4	103.0	149.0	212.0	263.5	309.4	358.0
converging value	0.0	45.0	112.0	158.0	225.0	270.0	315.0	132.0
starting value	0.0	51.5	97.4	143.2	194.8	257.8	303.6	271.0
converging value	0.0	45.0	90.0	135.0	180.0	247.0	293.0	140.0
starting value	0.0	45.8	91.6	137.5	189.0	252.0	297.9	197.0
converging value	0.0	45.0	90.0	135.0	180.0	247.0	293.0	142.0

Table 3.1 Summary of minimization -- 7 Blade cutter
(Speed 315 rpm, Model - 1)

	Blade angles							R(θ)
starting value	0.0	51.5	103.0	154.5	206.0	257.5	309.0	322.0
converging value	0.0	45.0	112.0	158.0	203.0	248.0	315.0	210.0
starting value	0.0	50.4	103.0	149.0	212.0	263.5	309.4	329.0
converging value	0.0	45.0	90.0	135.0	202.0	248.0	293.0	142.0
starting value	0.0	51.5	97.4	143.2	194.8	257.8	303.6	302.0
converging value	0.0	67.0	112.0	158.0	203.0	270.0	315.0	210.0
starting value	0.0	45.8	91.6	137.5	189.0	252.0	297.9	274.0
converging value	0.0	45.0	90.0	135.0	203.0	248.0	293.0	212.0

Table 3.2 - Summary of minimization--7 Blade cutter
(speed 315 rpm, feed 0.010 mm/tooth. MODEL -2)

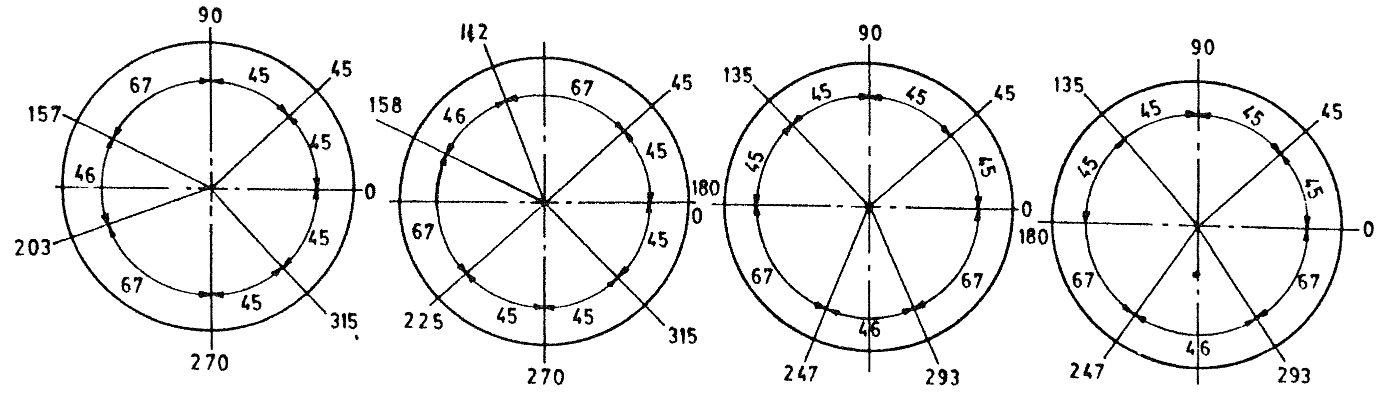


FIG. 3.3 FINAL BLADE LOCATIONS - RESULTS OF MINIMIZATIONS - MODEL 1

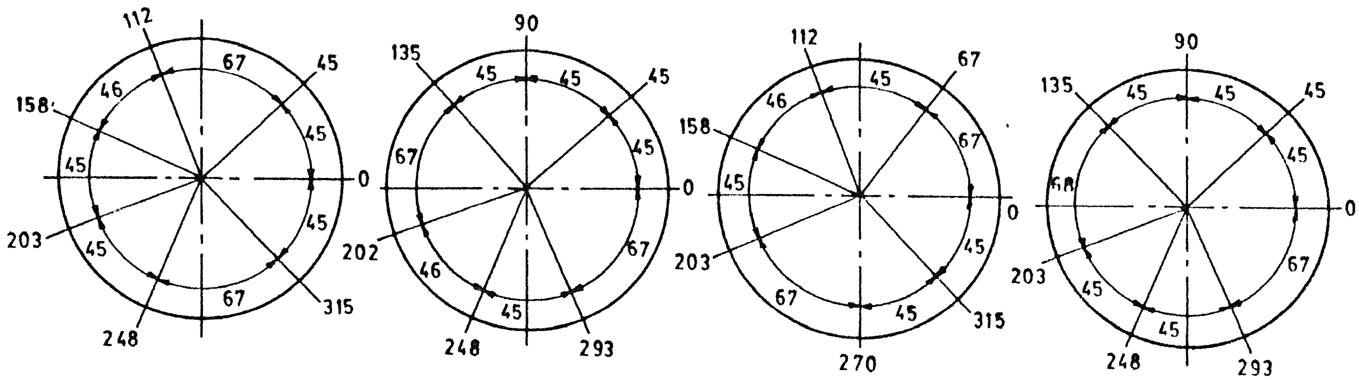


FIG. 3.4 FINAL BLADE LOCATIONS - RESULTS OF MINIMIZATIONS - MODEL 2

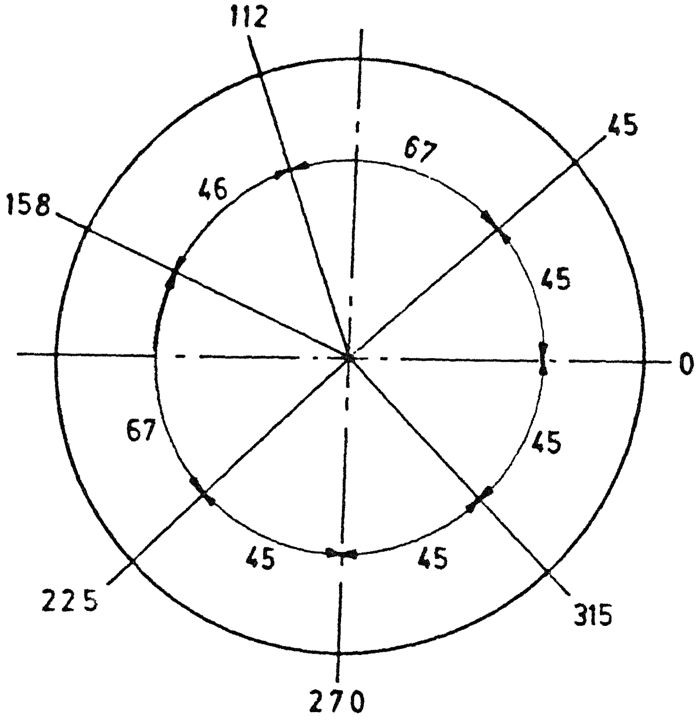


FIG. 3.5.1 BLADE LOCATIONS OF CUTTER DESIGN 1

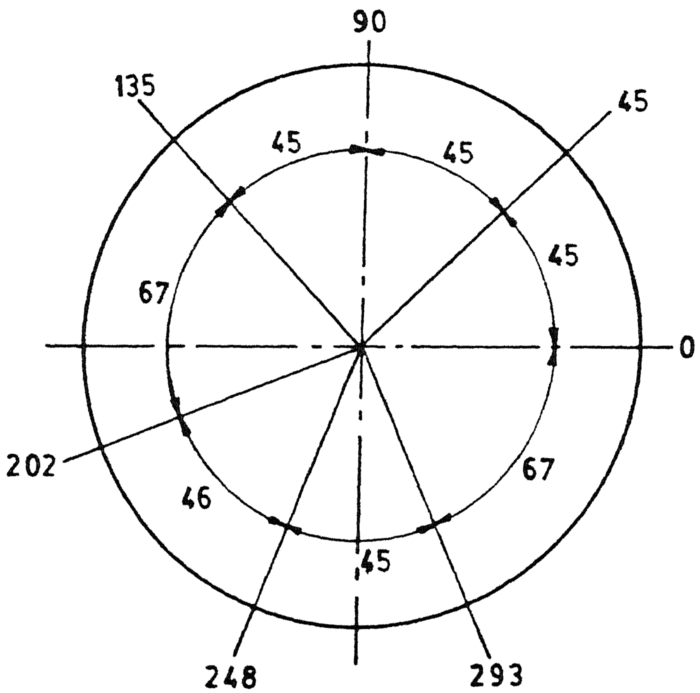


FIG. 3.5.2 BLADE LOCATIONS OF CUTTER DESIGN 2

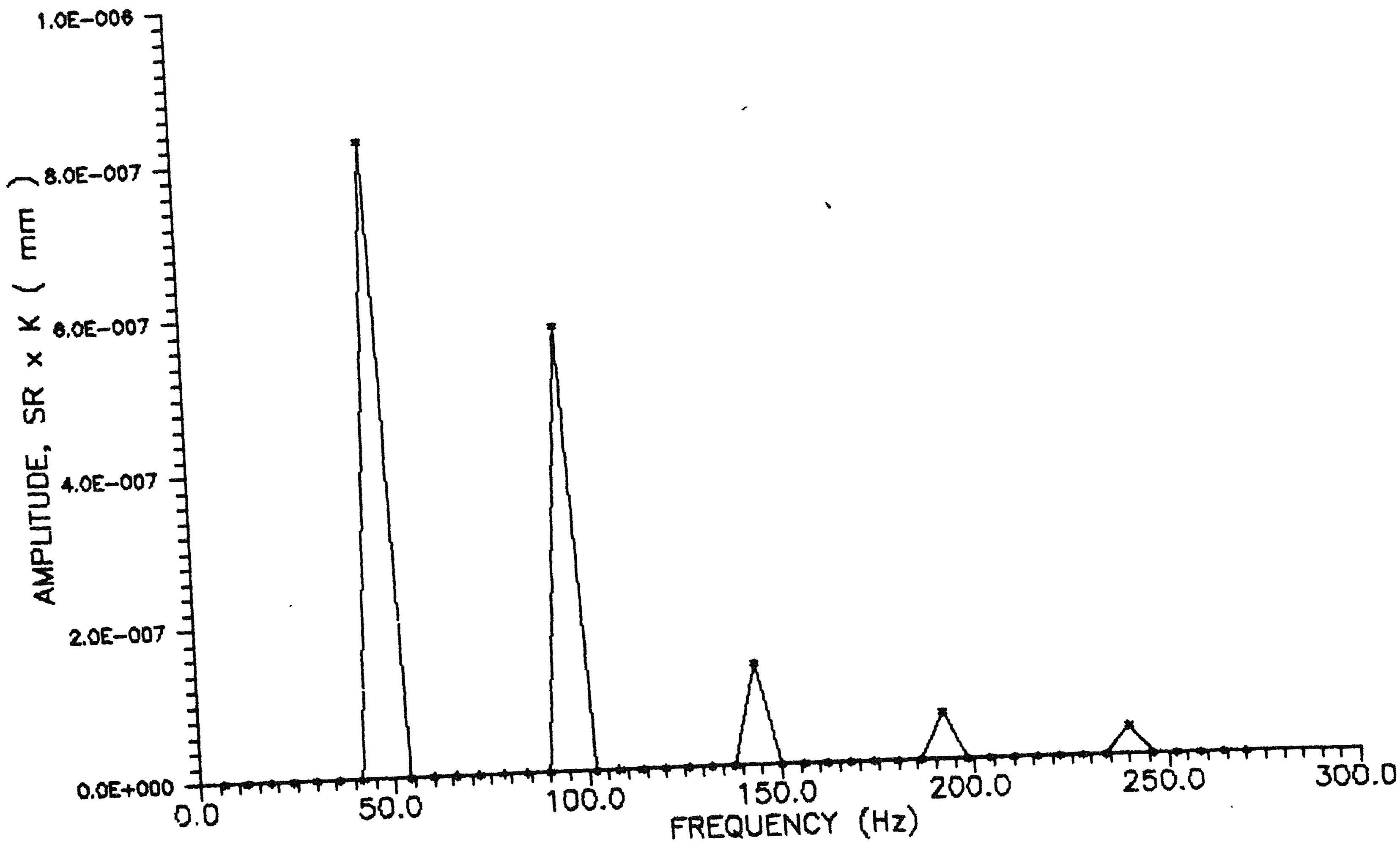


Fig. 3.6 Vibration Spectrum of Evenly Spaced Cutter with Eight Inserts, Model 1.

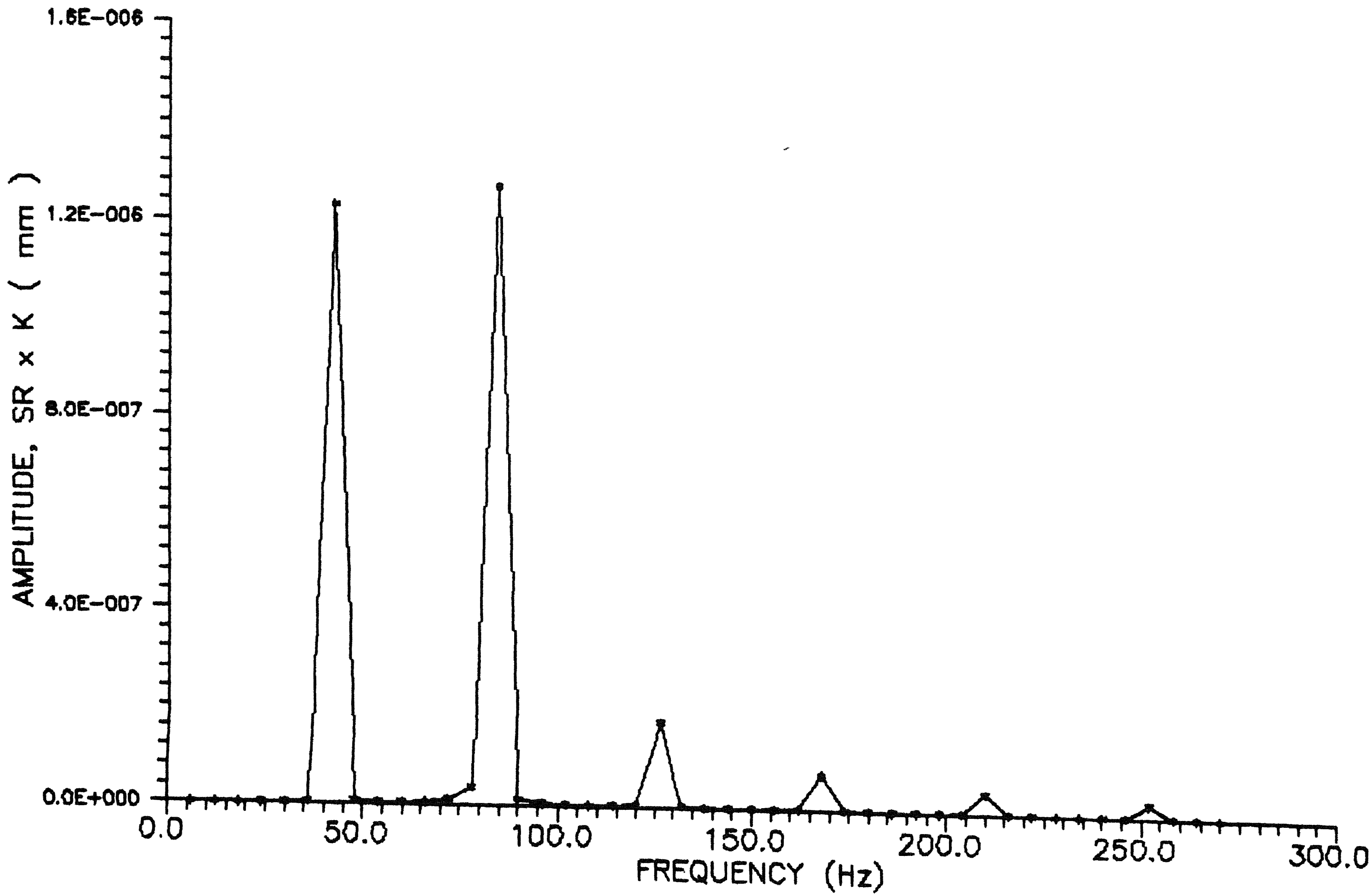


Fig. 3.7 Vibration Spectrum of Evenly Spaced Cutter with Seven Inserts, Model 1.

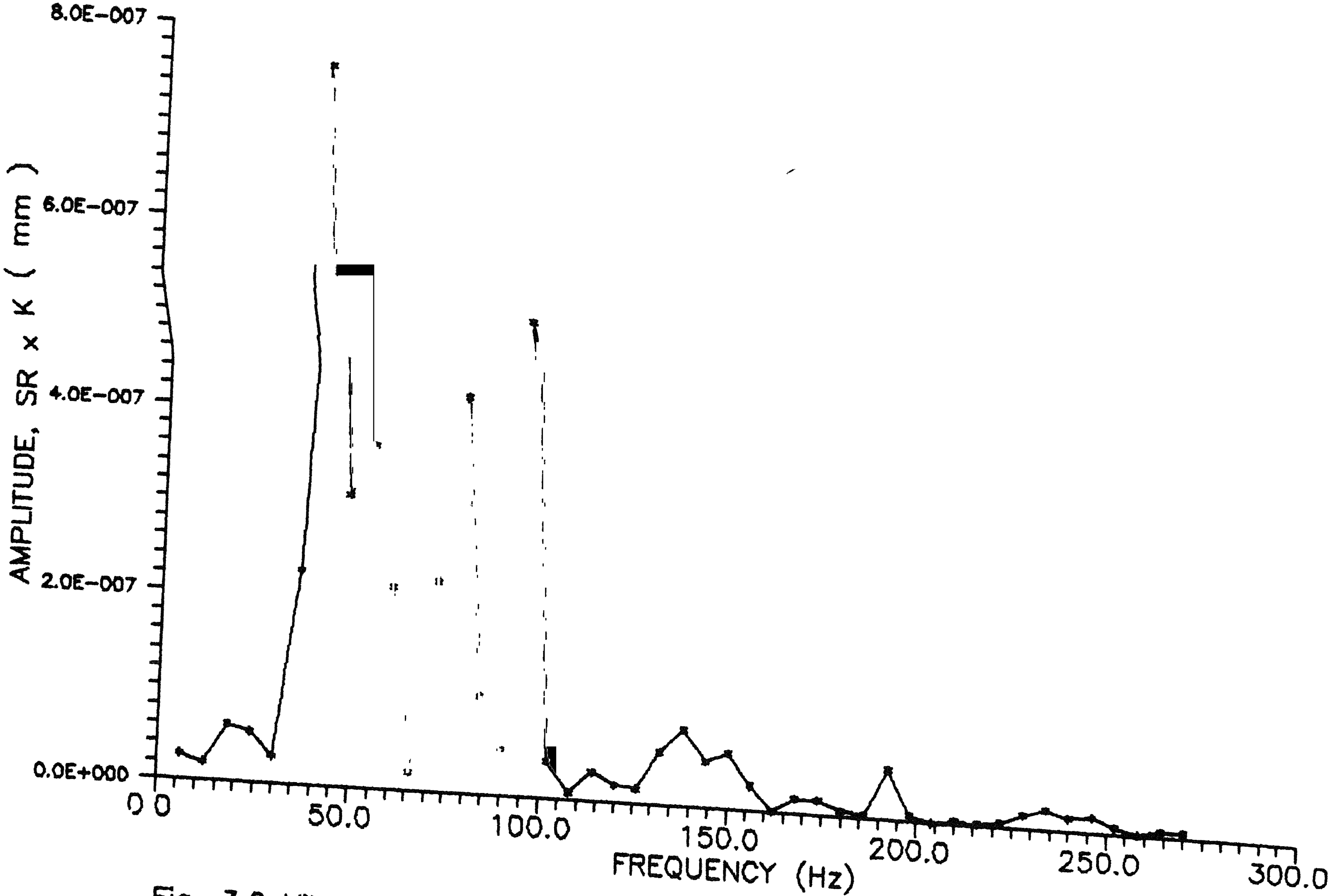


Fig. 3.8 Vibration Spectrum of Unevenly Spaced cutter Design 1 with Seven Inserts, Model 1.

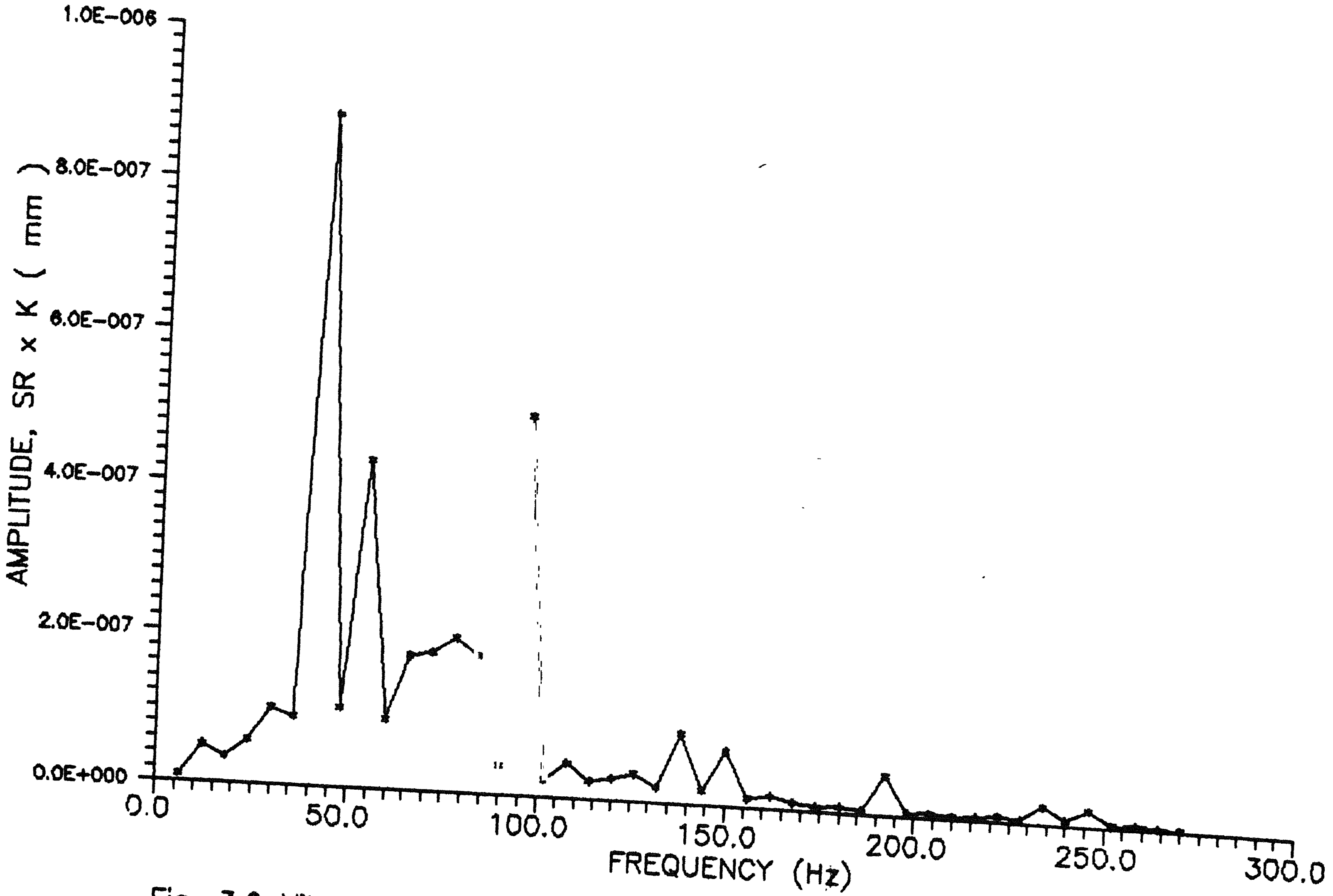


Fig. 3.9 Vibration Spectrum of Unevenly Spaced cutter Design 2 with Seven Inserts, Model 1.

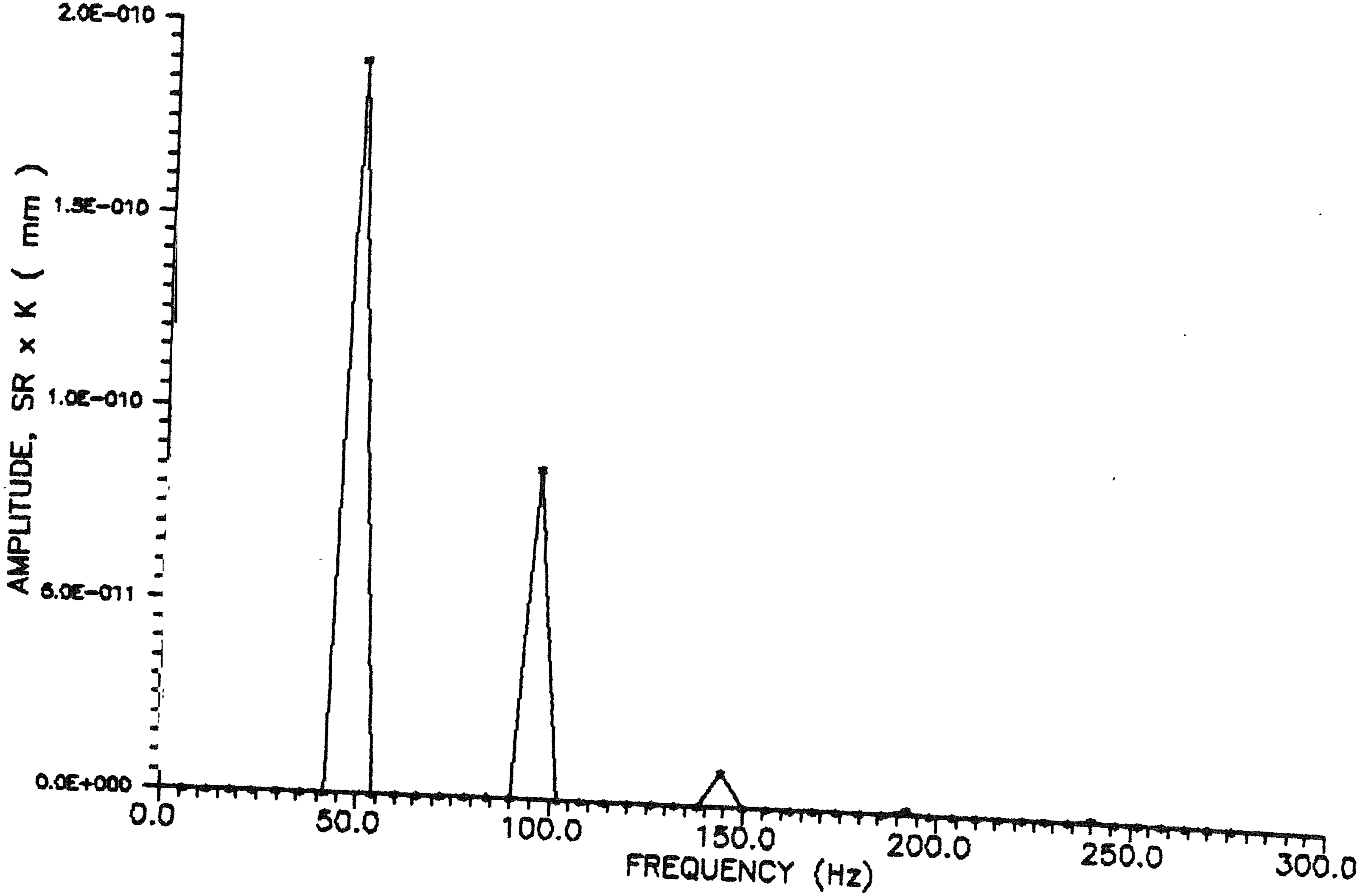


Fig.3.10 Vibration Spectrum of Evenly Spaced Cutter with Eight Inserts, Model 2.

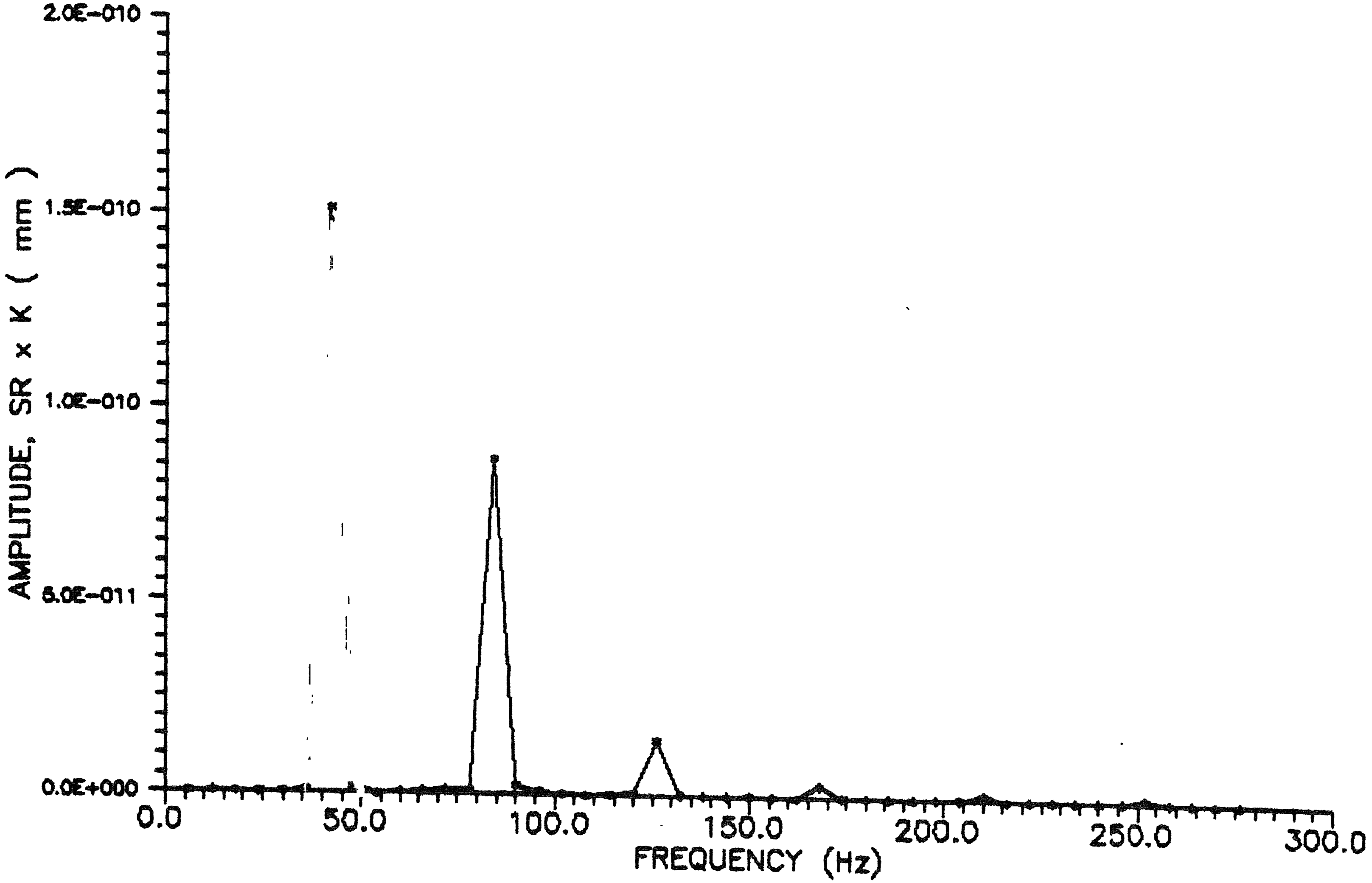


Fig.3.11 Vibration Spectrum of Evenly Spaced Cutter with Seven Inserts, Model 2.

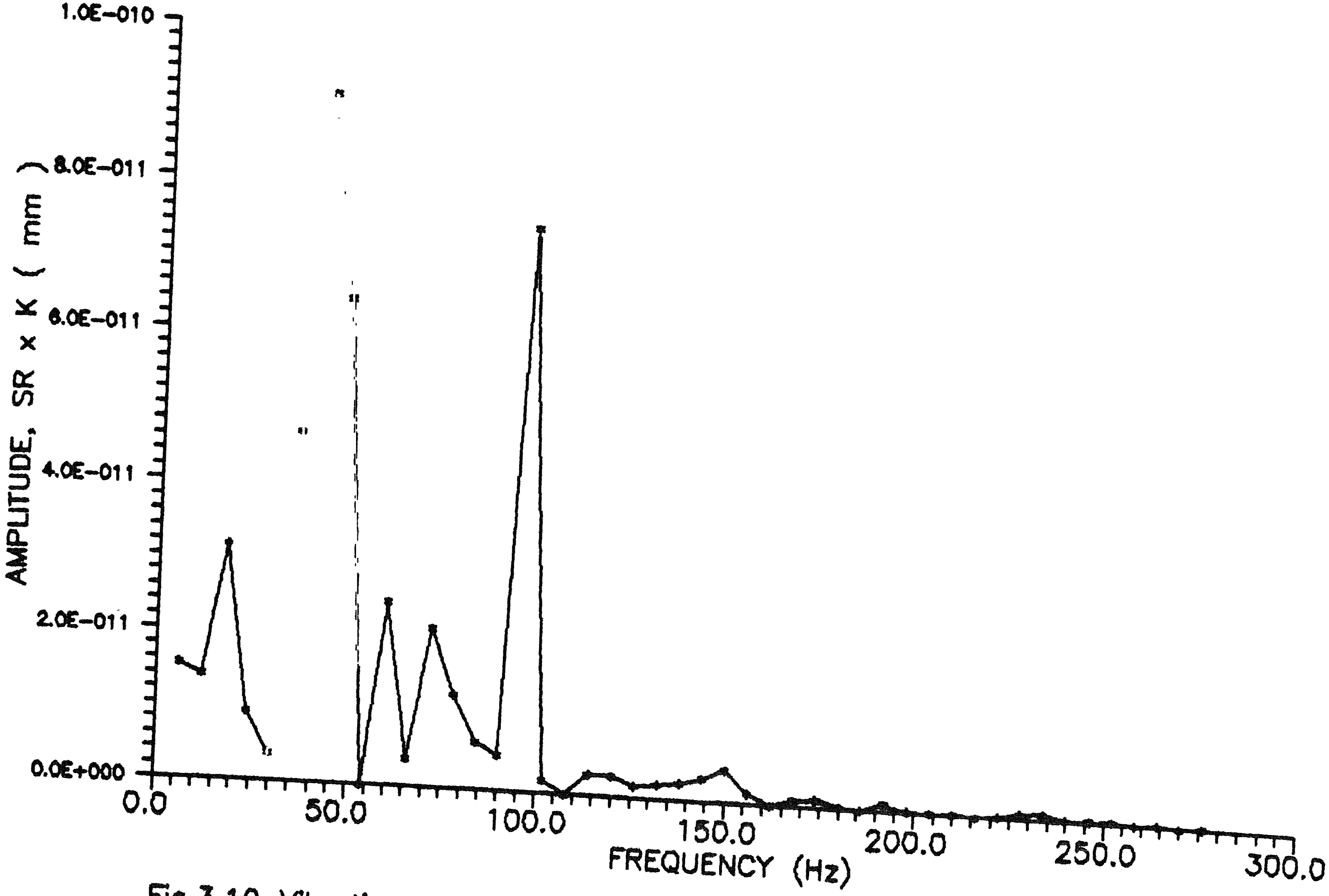


Fig.3.12 Vibration Spectrum of Unevenly Spaced Cutter Design 1 with Seven Inserts, Model 2.

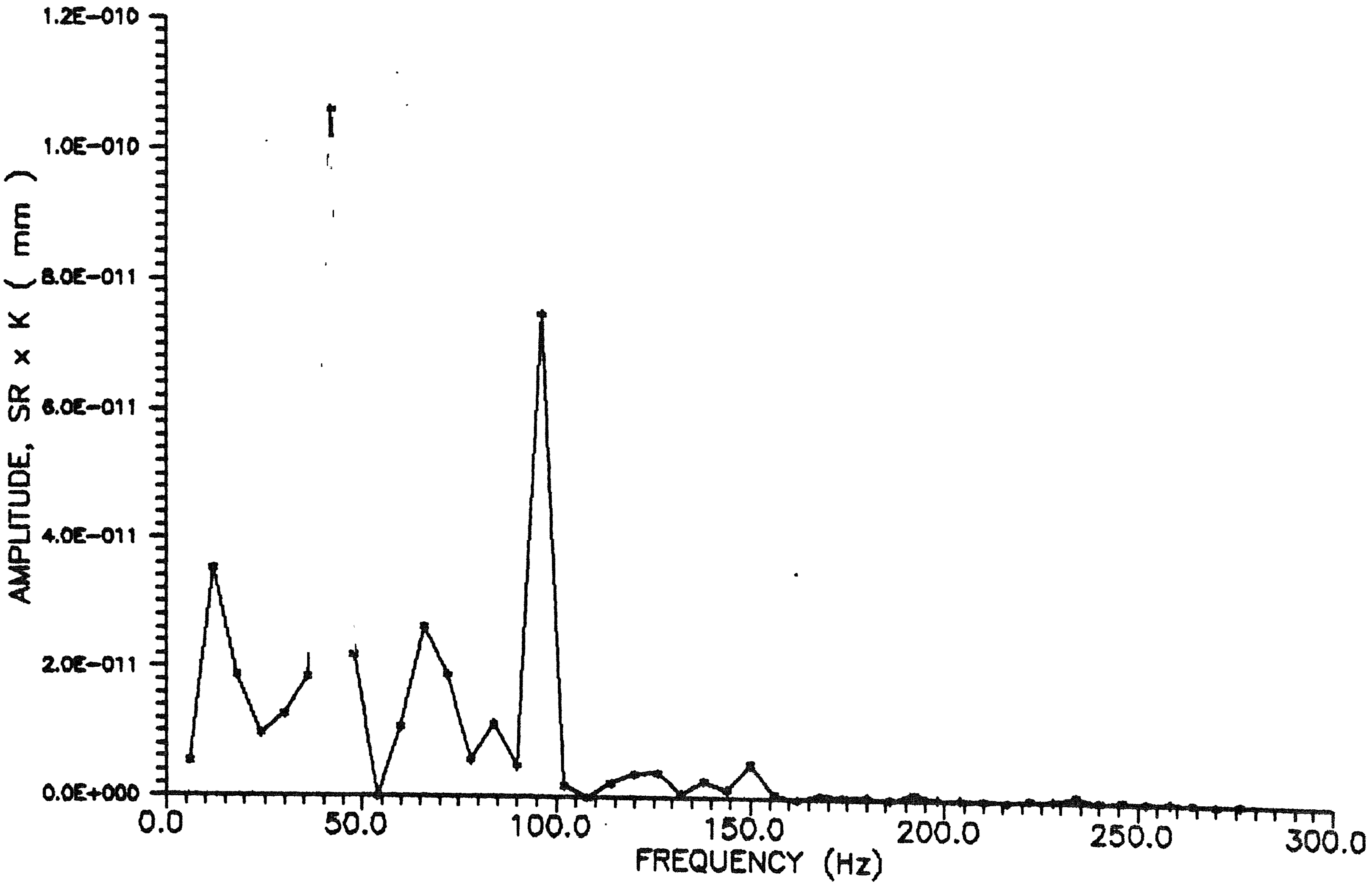


Fig.3.13 Vibration Spectrum of Unevenly Spaced Cutter Design 2 with Seven Inserts, Model 2.

	model - 1	model - 2
7 blades evenly spaced	-100%	+34%
7 blades un-evenly spaced	+ 52%	+52%
7 blades un-evenly spaced design -2	+ 57%	+56%

Table 3.3 Percentage reduction of the power of the vibration spectrum compared to evenly spaced 8 inserts cutt

	model - 1	model - 2
7 blades evenly spaced	+85%	+27%
7 blades un-evenly spaced	+91%	+53%
7 blades un-evenly spaced design -2	+89%	+45%

Table 3.4 Percentage reduction of the maximum vibration amplitude compared to evenly spaced 8 inserts cutte

CHAPTER 4

EXPERIMENTAL SET UP AND PROCEDURE

The aim of the experiment is to compare the effect of the designed non-uniform spacing inserts cutter with that of the standard cutter available with similar geometry. According to the theory explained in chapter 3, the non uniform inserts cutter will reduce the relative vibration between the cutter and the workpiece at a certain range of cutting parameters. Due to the constraints of the machine it may not be possible to maintain all the prescribed conditions mentioned for the designed cutter as such. Here, an attempt has been made to satisfy the conditions to the maximum possible.

The standard cutter available for study has 8 number of inserts which are evenly spaced (Make Sanvick-Model R 262.2-100-14). The cutter uses carbide tips as inserts. The diameter of the cutter is 100 mm and the details regarding the other dimensions are given in fig 4.1.

As per the angle specification obtained in design 1 (fig 3.5.1, chapter 3) a cutter has been fabricated. The material chosen for the cutter blank is high carbon high chromium steel. The dimensional details and drawing of the fabricated cutter are given in Fig 4.2. The new cutter is identical in all dimensions to that of the standard cutter except that of the tooth spacing. The spare parts used (like shim, shim screws, wedge screws etc.) are of the same type as that of the standard cutter. An overall view of the fabricated cutter is given in Fig. 4.4.

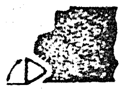
Since the optimum designs has achieved corresponds to a particular speed and feed range (i.e. 315 rpm, 0.010 mm/tooth - see Chapter 3), experiments are planned in such a way that its range of speed and feed includes the value of the optimum design. The range of cutting parameters chosen are given below:

1.	Speed = 315 rpm Feed = 30 mm/mt (0.010 mm/tooth)	depth of cut 0.1 mm 0.2 mm 0.3 mm
2.	Speed = 315 rpm Depth of cut = 0.2 mm	Feed 24 mm/mt 30 mm/mt 38 mm/mt
3.	Feed = 30 mm/mt Depth of cut = 0.2 mm	Speed 200 rpm 250 rpm 315 rpm

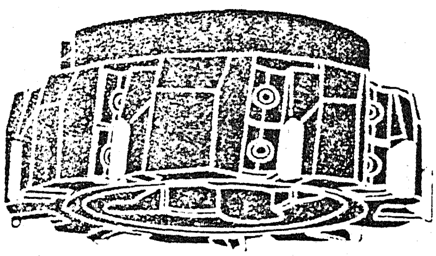
The material of the workpiece is EN-8 steel. The block (47 cm x 30 cm x 11 cm) is fixed rigidly to the table of the milling machine by means of clamps. The schematic diagram of the set-up is shown in Fig. 4.3.The overall view of the experimental set up is shown in fig.4.5. It was planned to analyse the vibration spectrum obtained during machining with the standard as well as the designed cutters by means of FFT analysers. But due to the unavailability of the equipment the experiments have to be limited to that of the time domain. A vibration pickup and a vibration meter (Model VM - 3314 A) are used for measuring the vibration. The pickup is of the electro magnetic type. The vibration meter can measure the displacement in the range of 0.01 to 1000 μ. The vibration pickup is fixed rigidly to the workprice.

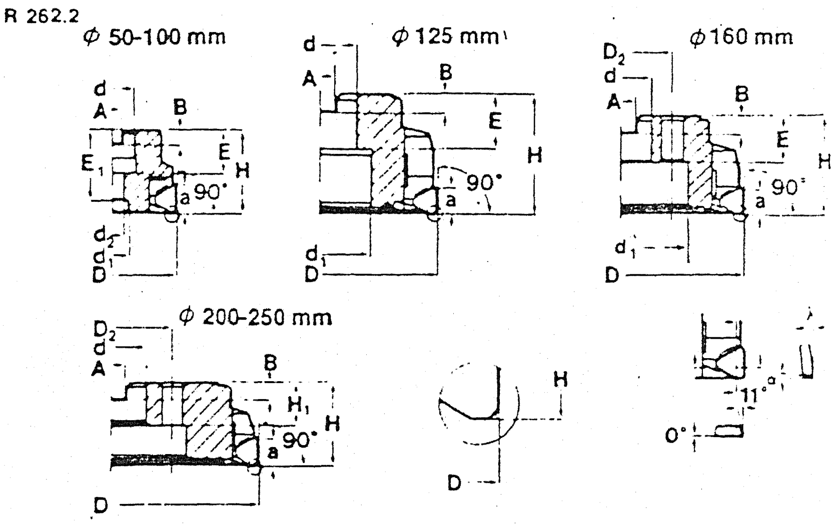
The experiments were carried out a horizontal milling machine (Model HMT- M3U). For analysis purpose the workpiece is divided into 9 parts and the vibrations of the cutting zone when the cutter passes each part is measured. The tests were conducted for both the standard and the design cutters in identical cutting conditions. Since the interest of study is in the Y direction (see Fig. 2.1 and Chapter 3) vibration measurements were carried out in that direction only. The experimental results are give in Table 5.1.

T-MAX square shoulder facemills

90°  R/L262.2

Positive rakes
ϕ50-250 mm



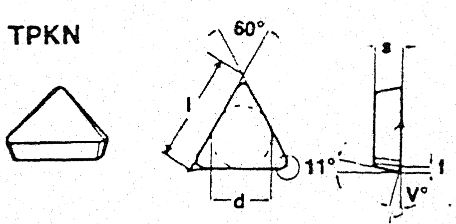


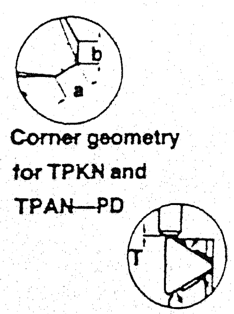
Dimensions

Ø	mm D	H	A	B	D_2	E	E_1	H_1	d	d_1	d_2	α°	λ°	a''	a''
100	100	50	14 4	8	–	25 5	33	–	32	27	18	7°	7°	–	18

Indexable inserts

TPKN





Tolerances, mm
Utility
TPKN
S = ± 0.025
T = ± 0.013

		ISO	mm l	d	s	a	b	f	v°
Utility	22	TPKN 2204 PD R TPKN 2204 PD L	22 0	12 70	4 76	1.4	0 7	0 40''	10''

FIG. T-MAX SQUARE SHOULDER FACE MILL - DETAILS.

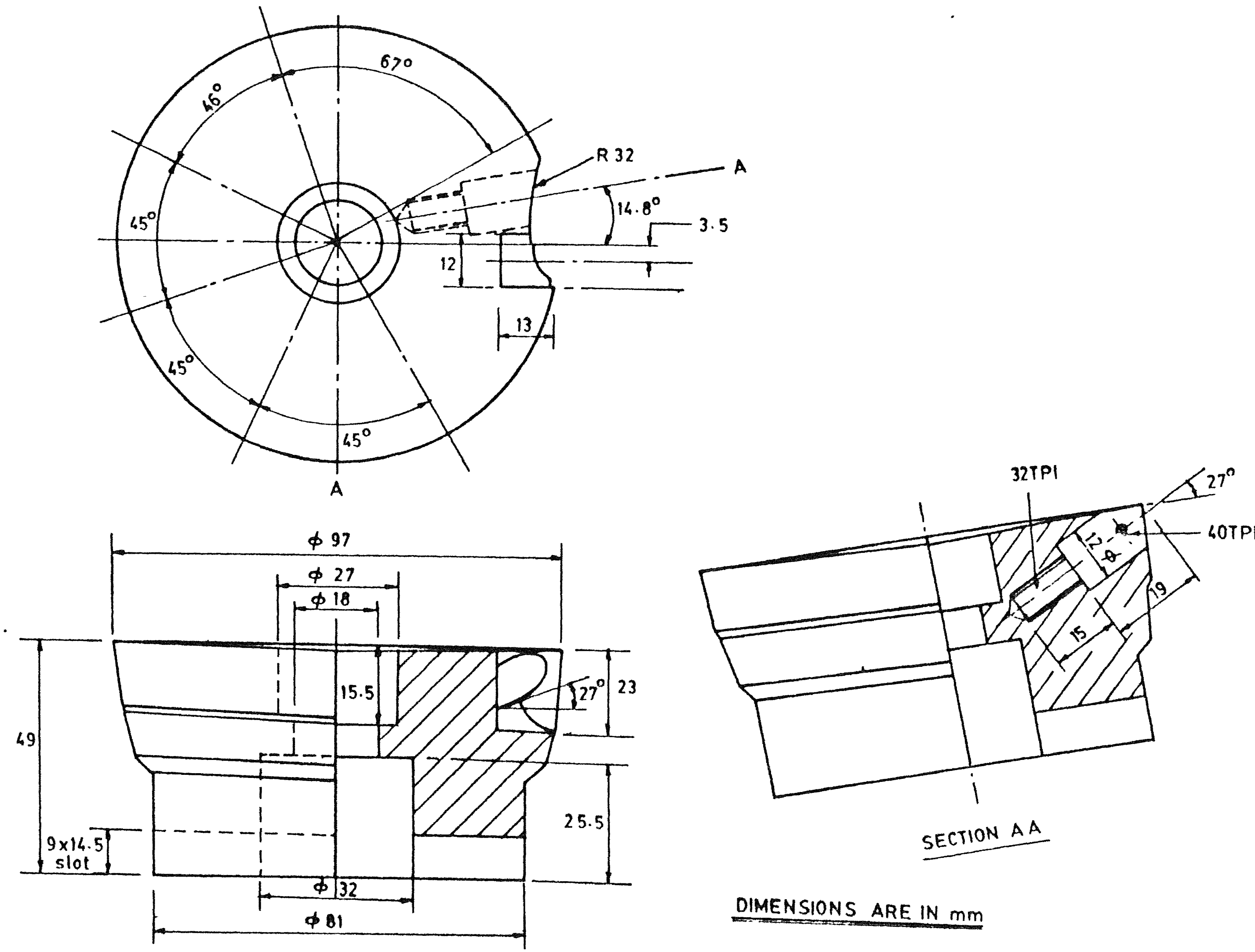


FIG. 4.2 DIMENSIONAL DETAILS OF THE FABRICATED CUTTER

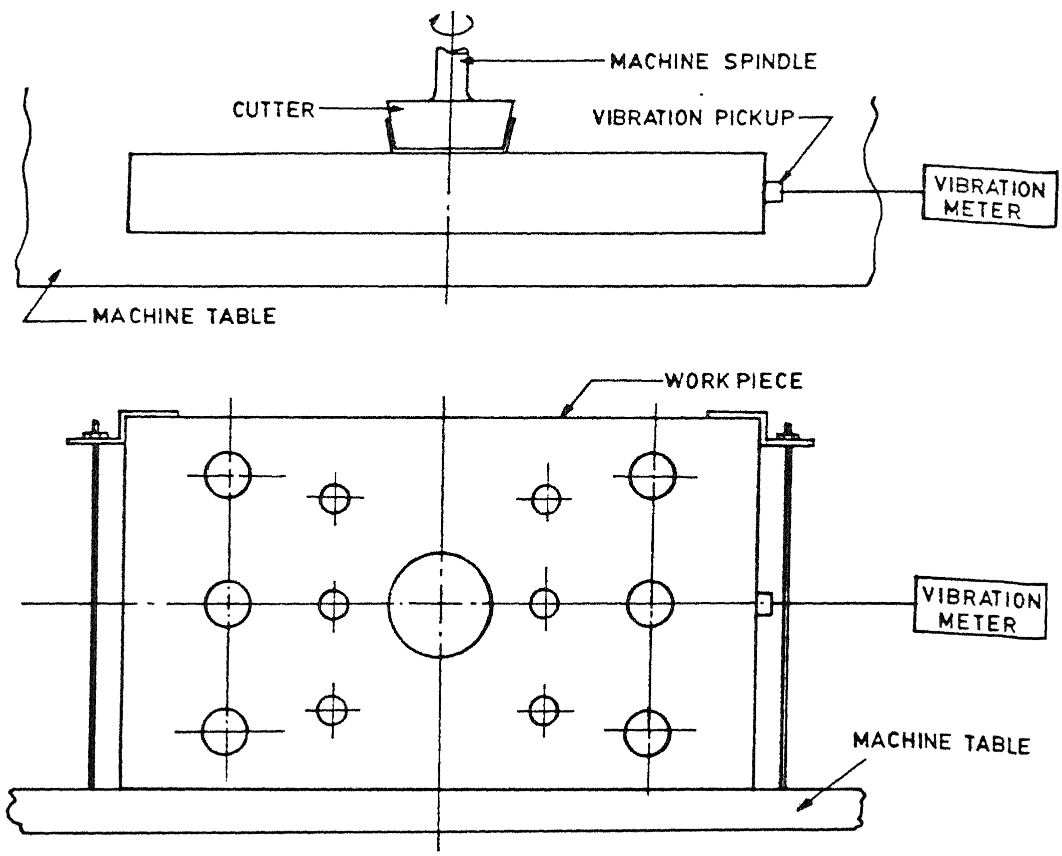


FIG. 4.3 SCHEMATIC DIAGRAM OF EXPERIMENTAL SETUP

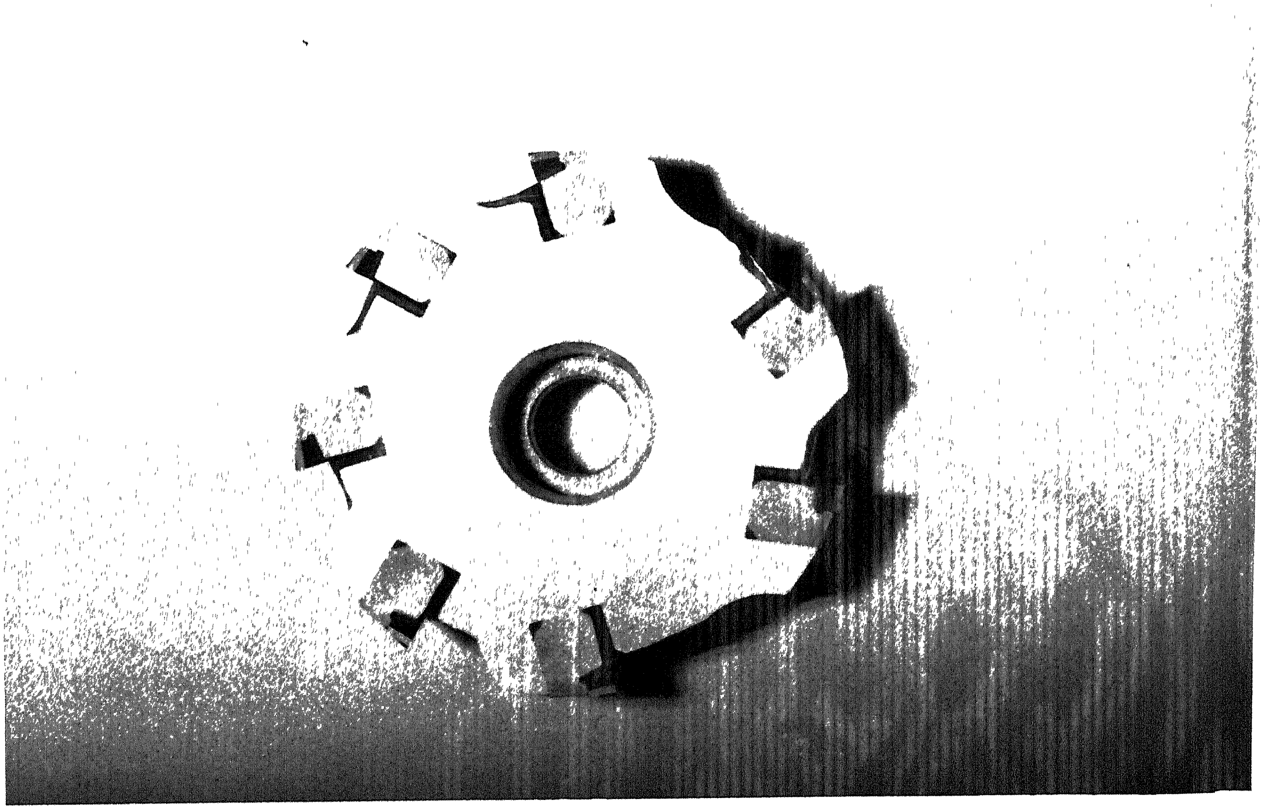


FIG. 4.4 OVERALL VIEW OF THE FABRICATED CUTTER.

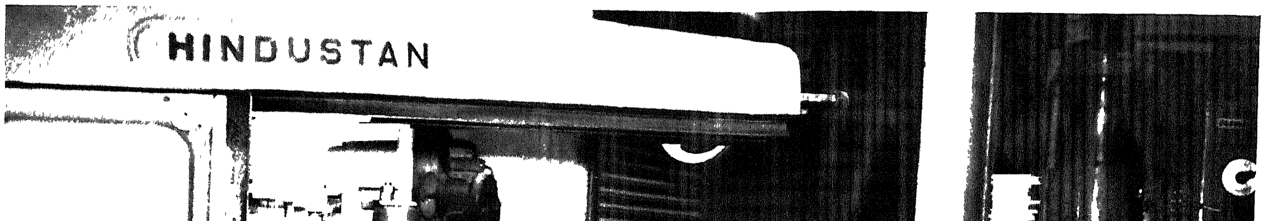


FIG. 4.5 OVERALL VIEW OF THE EXPERIMENTAL SETUP.

CHAPTER 5

EXPERIMENTAL RESULTS AND CONCLUSIONS

5.1 EXPERIMENTAL RESULTS AND DISCUSSIONS

A series of experiments were conducted to determine the performance of the face milling cutter fabricated. Experiments were conducted in the various speed and feed ranges as explained in Chapter 4. The amplitude of the vibration obtained during machining at different positions of the workpiece for the newly designed cutter is given in Table 5.1. The vibration amplitude for the standard cutter with 8 inserts are given in Table 5.2.

For making a comparative study,the experimental results are plotted at different speeds, feed and depth of cuts as shown in Fig. 5.1 - 5.3. From the graphs it is clear that the vibration amplitude is always less for the newly designed cutter. To analyse the effect of the vibration reductions at different cutting conditions, a vibration reduction factor can be defined as

$$\frac{1}{K} = \frac{\text{Amplitude of vibration by standard cutter}}{\text{Amp. of vibration with designed cutter}}$$

where,K is the vibration reduction factor.

A Table has made (table 5.3)which gives the value of K at different conditions. It can be seen that the vibration reduction factor is more in the cae when feed is 30 mm/mt and depth of cut 0.2 mm, which indicates the effect of thedesignedcutter is more predominant in that cutting conditions. The value of K is fairly

high in the designed cutting conditions, hence it proves the effectiveness of the designed cutter in reduction of vibration.

5.2 CONCLUSIONS

A face milling cutter with uneven spacing has been designed and fabricated. The newly designed cutter is investigated at different cutting conditions to find out the effect of the new cutter in reduction of vibration. A standard cutter with evenly spaced 8 inserts was used for comparison.

It has been proved that the uneven spacing cutters reduces the vibration in the cutting zone at a certain range of cutting conditions. The reduction is due to redistribution effect caused by the uneven impacts and hence it avoids the system to exite at its natural frequencies.

5.3 SCOPE FOR FUTURE WORK

Analytical expressions for vibration in face milling is not easily obtainable because of the following factors. The machining process is complicated due to the nature of multi-tooth cutting, variable chip loading, variable forcing functions direction, and varying workpiece geometry. Hence a simulation model may be a better and viable approach to the predictions of vibrations during machining. For making a simulation model a dynamic force model can be developed which will predict the instantaneous cutting forces and which also should characterize the structural dynamics. The model can include a feed back system to make it a closed loop one,so that the instantaneous changes cutting

parameters are can be taken care of. The natural frequencies of the milling machine structure can be found more accurately if the number of elements increased considerably and the different types of elements can be tried for idealization. For experimentally analysing the cutting process a spectrum analyser with FFT can be used and the experiments can be tried at wider range of various cutting conditions.

	Positions	1	2	3	4	5	6	7	8	9
Speed 315 rpm feed 30 mm/mt	depth of cut 0.1 mm	13	13	15	15	17	19	18	25	25
	depth of cut 0.2 mm	13	14	16	16	18	22	20	26	27
	depth of cut 0.3 mm	16	20	23	26	27	28	28	32	33
Speed 315 rpm depth of cut 0.2 mm	feed 24 mm/mt	13	13	14	14	16	19	20	22	25
	feed 30 mm/mt	13	14	16	16	18	22	20	26	27
	feed 38 mm/mt	15	20	23	25	26	27	25	34	36
Feed 30 mm/mt depth of cut 0.2 mm	speed 200 rpm	13	21	23	25	27	27	27	30	30
	speed 250 rpm	14	20	22	22	23	25	24	29	29
	speed 315 rpm	13	14	16	16	18	22	20	26	27

Table 5.1 Experimental results : Absolute vibration (microns) obtained at various positions of workpiece. (with Designed Cutter)

	Positions	1	2	3	4	5	6	7	8	9
Speed 315 rpm feed 30 mm/mt	depth of cut 0.1 mm	13	16	20	20	22	22	18	32	32
	depth of cut 0.2 mm	15	18	21	21	23	24	22	33	33
	depth of cut 0.3 mm	18	24	25	27	28	30	30	40	34
Speed 315 rpm depth of cut 0.2 mm	feed 24 mm/mt	13	18	19	19	18	21	22	33	34
	feed 30 mm/mt	15	18	21	21	23	24	22	33	33
	feed 38 mm/mt	16	20	30	31	30	32	32	38	40
Feed 30 mm/mt depth of cut 0.2 mm	speed 200 rpm	16	26	29	29	30	32	33	38	38
	speed 250 rpm	16	22	24	26	28	29	30	37	38
	speed 315 rpm	15	18	21	21	23	24	22	33	33

Table 5.2 Experimental results : Absolute vibration (microns) obtained at various positions of workpiece. (with Standard Cutter)

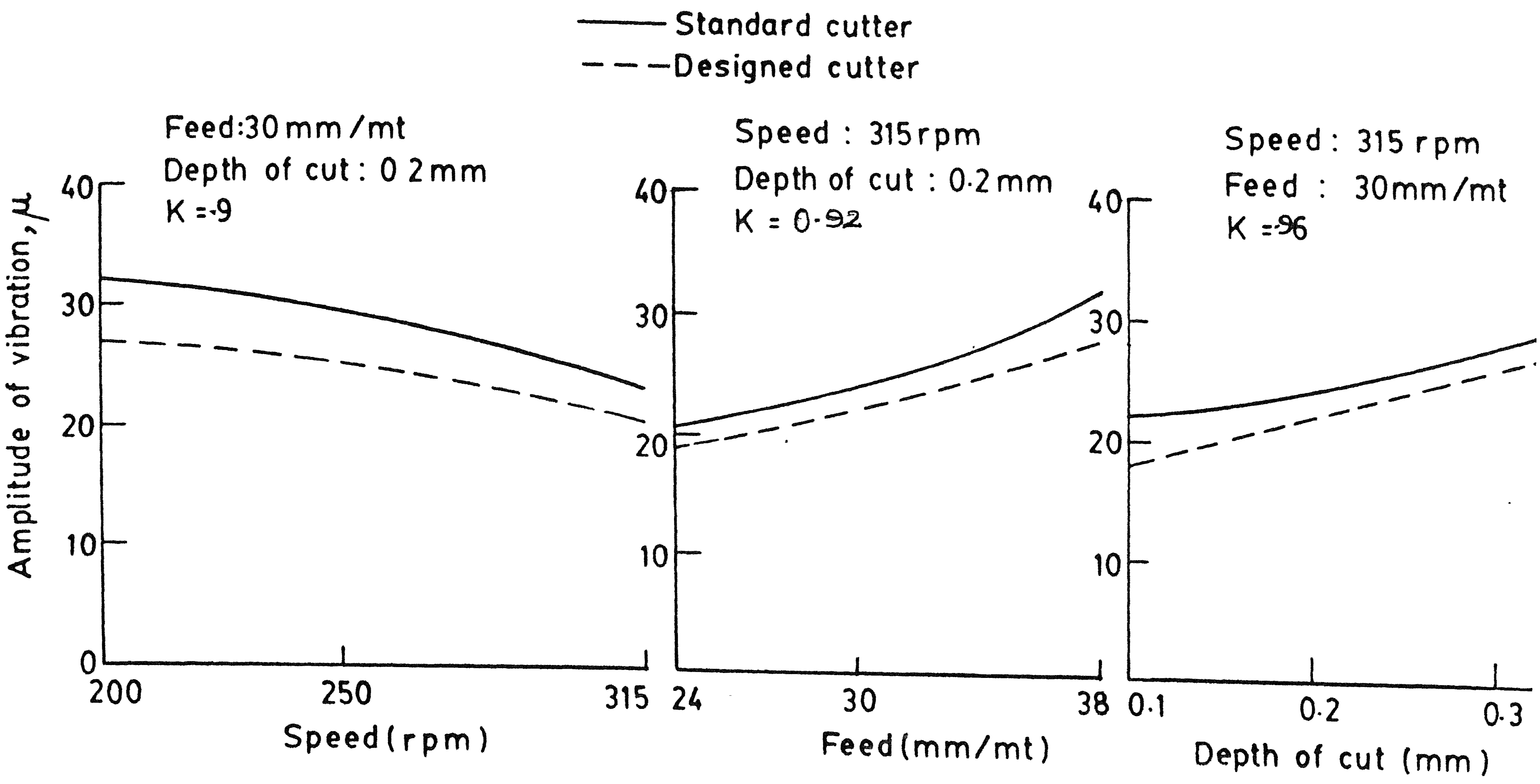


FIG. 5·1 VIBRATIONS SPECTRA (Position 6)

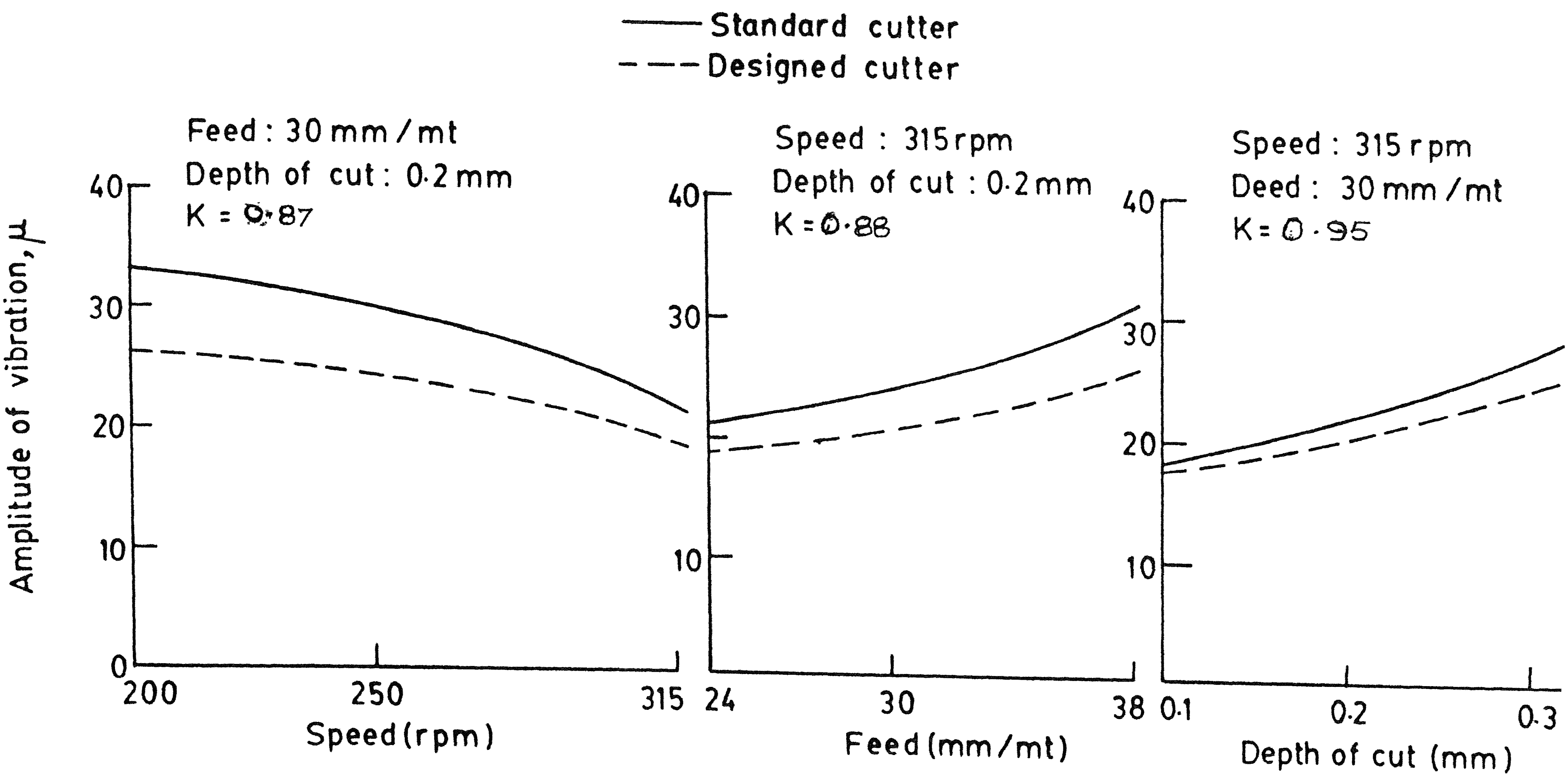


FIG. 5.2 VIBRATION SPECTRA (Position 7)

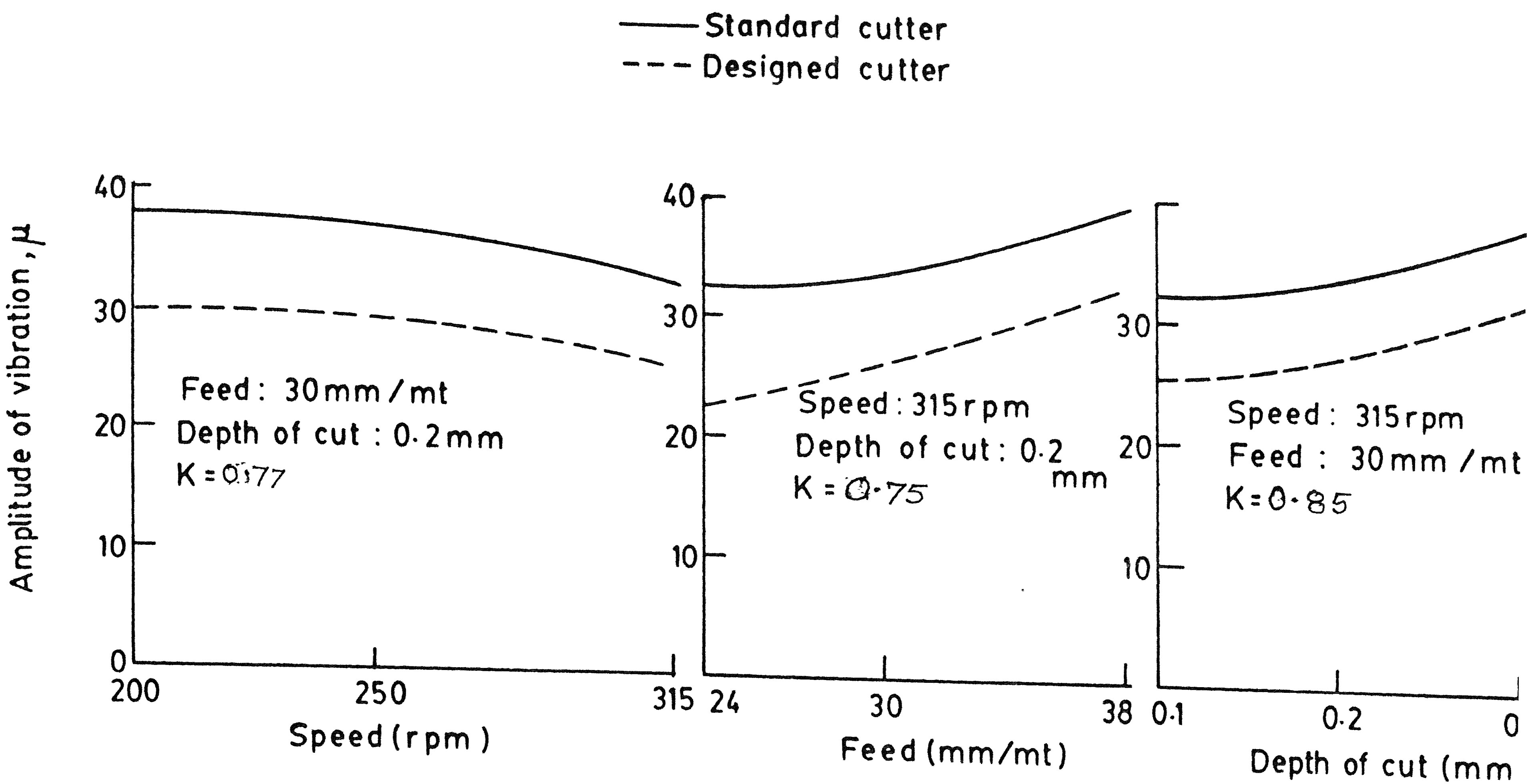


FIG. 5.3 VIBRATION SPECTRA (Position 8)

measuring position of vibration zone	feed 30 mm/tooth depth of cut 0.2mm speed, varies	speed 315 rpm depth of cut 0.2mm feed, varies	speed 315 rpm feed 30 mm/ tooth dep:cut, varies
(at position 6)	0·9	0·92	0·96
(at position 7)	0·87	0·88	0·95
(at position 8)	0·77	0·75	0·85

Table5.3 values vibration reduction factor (K)

at different cutting conditions

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